Although a related notion is considered in BARENDREGT [1981], from our point of view this principle seems rather more dubious than, say, (BETA): surely one ought to distinguish the order of steps in making a computation, and not merely the steps themselves. However, all the various naturality and coherence conditions suitable for sections 3,4 do seem to require (beta comm).

A.2 For the record, the coherence conditions referred to are the following. We suppose ι decreasing, as in section 3, and use the notation there for objects, morphisms, and 2-cells.

(For γ,ι):

$$\gamma(id(A),a)\cdot\iota(A)F(a) = id(F(a))$$
$$\gamma(a,id(B))\cdot F(a)\iota(B) = id(F(a))$$
$$\gamma(ab,c)\cdot\gamma(a,b)F(c) = \gamma(a,bc)\cdot F(a)\gamma(b,c)$$
$$\gamma(a',b')\cdot F(p)F(r) = F(pr)\cdot\gamma(a,b)$$

(and similarly for G.)

(For k): $k(A,id(B))\cdot\underline{A}(A,\iota(B))K(A,B) = K(A,B)$

$$k(A,b'b)\cdot\underline{A}(A,\gamma(b',b))K(A,B_2) = k(A,b')\underline{B}(FA,b)\cdot\underline{A}(A,Gb')k(A,b)$$

$$k(A,b')\cdot\underline{A}(A,G(p))K(A,B) = K(A,B_1)\underline{B}(FA,p)\cdot k(A,b)$$

(and similarly for l.)

(For η): $k(A,b)L(A,B)\cdot\underline{A}(A,Gb)\eta(A,B) = K(A,B_1)l(A,b)\cdot\eta(A,B_1)\underline{A}(A,Gb)$

(and similarly for ϵ; these give the "laxity" of the modifications η,ϵ.)

A.3 Similar conditions apply for the situation of section 4, with increasing ι.

(For e): $e(id(B)) = \beta(B)\iota(B)$
$$e(b'b)\cdot\beta(B_2)\gamma(b',b) = b'e(b)\cdot e(b')FG(b)$$
$$p\beta(B)\cdot e(b) = e(b')\cdot\beta(B_1)FG(p)$$

(and similarly for n; note the similarity with the conditions for k,l.)

(for ρ): $\rho(A)F(a)\cdot\beta(F(A))F(n(a)) = F(a)\rho(A_1)\cdot e(F(a))F(\alpha(A_1))$

(and similarly for σ; these give the "laxity" of the modifications ρ,σ.)

REFERENCES

H.P. BARENDREGT, The Lambda Calculus, (North-Holland, 1981).

J.-Y. GIRARD, The system F of variable types, fifteen years later, Theoretical Comp. Sci. 45 (1986) 2, 159-192.

J.W. GRAY, Formal Category Theory: Adjointness for 2-categories, (Springer LNM 391, 1974).

G.M. KELLY, ed., Category Seminar, (Springer LNM 420, 1974).

G.M. KELLY, R. STREET, Review of the elements of 2-categories, in KELLY [1974], 75-103.

G.M. KELLY, Basic Concepts of Enriched Category Theory, London Math. Soc. Lecture Note Series 64, (Cambridge University Press, 1982).

J. LAMBEK, P.J. SCOTT, Introduction to Higher Order Categorical Logic, (Cambridge University Press, 1986).

D.S. SCOTT, Continuous lattices, in F.W Lawvere, ed., Toposes, Algebraic Geometry and Logic, (Springer LNM 274, 1972).

D.S. SCOTT, Data types as lattices, SIAM J. Comput. 5 (1976) 522-587.

R.A.G. SEELY, Hyperdoctrines and Natural Deduction, Ph.D. Thesis, (University of Cambridge, 1977).

R.A.G. SEELY, Weak adjointness in proof theory, in M.P. Fourman, C.J. Mulvey, and D.S. Scott, eds., Applications of Sheaves, (Springer LNM 753, 1979), 697-701.

R.A.G. SEELY, Categorical semantics for higher order polymorphic lambda calculus, Math. Report 86-2, (McGill University 1986). (To appear in J. Symbolic Logic, 1987.)

Proceedings of the Symposium on Logic in Computer Science, 1987
(Computer Society of the IEEE 1987) 65–71