Infinite Series — Notes for Cal II

Test	Series	Conditions	Conclusion	Comments	Examples
GS	$\sum ar^n$	$\lvert r\rvert < 1$ $\lvert r\rvert \geq 1$	C D	sum: $\frac{a}{1-r}$	$\sum \frac{5^n+3^n}{7^n}$ $\sum \frac{8^n}{7^n}$
p S	$\sum 1/n^p$	$p > 1$ $p \leq 1$	C D		$\sum \frac{1}{n\sqrt{n}}$ $\sum \frac{1}{\sqrt[3]{n}}$
nTT	$\sum a_n$	$\lim a_n \neq 0$	D	fails if $\lim a_n = 0$	$\sum \frac{2n^3-3n}{3n^3+n^2-1}$ **not:** $\sum \frac{1}{\ln n}$
Tests for Positive Term Series					
$\int$ T	$\sum a_n$	f cont, $\searrow$, ≥ 0, $a_n = f(n)$ $\int_*^\infty f(x)\,dx$ C $\int_*^\infty f(x)\,dx$ D	 C D	 *i.e.* $\int, \sum$ C or D together	 $\sum \frac{n}{e^{n^2}}$ $\sum \frac{\ln n}{n}$
CT	$\sum a_n$	$0 \leq a_n \leq c_n$, $\sum c_n$ C $0 \leq d_n \leq a_n$, $\sum d_n$ D	C D	"less than C is C" "bigger than D is D"	$\sum \frac{1}{2^n+n}$ $\sum \frac{1}{\ln n}$
LCT	$\sum a_n$	$\lim \frac{a_n}{b_n}$ exists, (finite), $\neq 0$ $\sum b_n$ C (lim $\frac{a_n}{b_n} = 0$ ok) $\sum b_n$ D (lim $\frac{a_n}{b_n} = \infty$ ok)	 C D	 *i.e.* comparable series C or D together	 $\sum \frac{n-10}{n^3+2n^2-1}$ $\sum \frac{\sqrt{n}+3}{n+7}$
Tests for Arbitrary (positive and negative term) Series					
AST	$\sum(-1)^n a_n$	$a_n \geq 0$, $\{a_n\} \searrow$, $\lim a_n = 0$	(C)C	remainder $\lvert R_N\rvert \leq a_{N+1}$	$\sum(-1)^n \frac{\sqrt{n}}{n+2}$
RT	$\sum a_n$	$\lim \lvert\frac{a_{n+1}}{a_n}\rvert < 1$ $\lim \lvert\frac{a_{n+1}}{a_n}\rvert > 1$	AC D	fails if $\lim \lvert\frac{a_{n+1}}{a_n}\rvert = 1$	$\sum \frac{n!}{n^n}$ $\sum \frac{(2n)!}{4^n}$ **not:** $\sum \frac{n(n+2)}{\sqrt{n+1}(n^2+4)}$
$\sqrt[n]{}$T	$\sum a_n$	$\lim \sqrt[n]{\lvert a_n\rvert} < 1$ $\lim \sqrt[n]{\lvert a_n\rvert} > 1$	AC D	fails if $\lim \sqrt[n]{\lvert a_n\rvert} = 1$	$\sum \frac{1}{(\ln n)^n}$ $\sum (\frac{2k}{k+2})^n$ **not:** $\sum (1+\frac{1}{n})^n$

Flow Chart for AC, CC, D: