

Cal II (S) (Maths 201-NYB)

(Marks)

Please show all your work, so as to justify your answers. Answers without justification will not receive full credit. Presentation is important, and some credit will be lost for messy or incoherent work.

1. For each of the following sequences, does the sequence converge? And if so, find its limit as (2×2)

(a) $\left\{n\sin\left(\frac{1}{n}\right)\right\}$

(b) $\left\{n-\sqrt{n}\right\}$

2. For each of the following series, determine whether or not it converges, and if it does, find the (2×3) sum of the series.

(a) $\sum_{n=0}^{\infty} \left(1 - \frac{3}{n}\right)^n$

(b) $\sum_{n=1}^{\infty} \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right)$

3. Determine whether each of the following series converges or diverges. State the tests you use, (16)and verify that the conditions for using them are satisfied.

(a) $\sum_{n=0}^{\infty} \sin\left(\frac{1}{n^2}\right)$

(b) $\sum_{n=0}^{\infty} \frac{(2n)!}{2^n n!}$ (d) $\sum_{n=0}^{\infty} \frac{5^{2n}}{n^n}$

(c) $\sum_{n=0}^{\infty} \frac{\ln n}{n^2}$

(10)4. Classify each of the following series as absolutely convergent, conditionally convergent or divergent. (Justify your conclusions.)

(a) $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{2^n}{n^2}$ (b) $\sum_{n=0}^{\infty} (-1)^n \frac{n+1}{\sqrt[4]{2n^9+6n+1}}$

- 5. Determine the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(x+2)^n}{4^n \sqrt{n^2+1}}$. (5)
- 6. Suppose that $\sum_{n=0}^{\infty} a_n$ converges, $a_n \ge 0$ for all $n \ge 0$. (3)

(a) What is $\lim_{n\to\infty} a_n$? (b) Does $\sum_{n=0}^{\infty} \frac{n a_n}{2n+1}$ converge?

Be sure to (briefly!) justify your answers (e.g. mention which theorem or convergence criteria you are using).

7. Find the Maclaurin series for $f(x) = \frac{x}{x+1}$. Write down the first four non-zero terms explicitly, (6)and give a general formula for the series. What is the interval of convergence for this series?

(Total: 50)