Instructor: Dr. R.A.G. Seely

Test 2
(version for practice)

Cal II (S) (Maths 201–NYB)

(Marks)

Justify *all your answers—just having the correct answer is not sufficient.*
Presentation is important, and some credit will be lost for messy or incoherent work.
Pace yourself—a rough guide is to spend not more than $2m$ minutes or so on a question worth m marks.

(3×3) 1. Evaluate the following limits:

(a) $\lim_{x\to+\infty} \left(\sqrt{x^2+3x} - x\right)$ (b) $\lim_{x\to 0} \dfrac{\ln(1-2x)}{\tan x}$ (c) $\lim_{x\to 0} \left(1-\frac{x}{3}\right)^{\frac{4}{x}}$

(2×4) 2. Determine whether these improper integrals converge or diverge: if an integral converges, give the exact value of the integral.

(a) $\displaystyle\int_0^2 \frac{dx}{(x-1)^2}$ (b) $\displaystyle\int_0^\infty x\,\mathrm{e}^{-x^2}\,dx$

(3×5) 3. Let $\mathcal{R}$ be the three-sided region in quadrant I between the y-axis and the curves $y=\sin x$ and $y=\cos x$.

(a) Find the area of $\mathcal{R}$.

(b) Find the volume of the solid obtained when $\mathcal{R}$ is rotated about the line $x=-1$.

(c) Find the volume of the solid obtained when $\mathcal{R}$ is rotated about the x-axis.

(4) 4. Find the length of the curve $y=\frac{1}{8}x^2-\ln x$, from $x=1$ to $x=2$.

(5) 5. Find the general solution of the differential equation: $x\,y'+1=y^2$ Express y as a function of x, or x as a function of y (your choice!), using an appropriate constant of integration as necessary.

(3) 6. For the differential equation $P'=P(10-P)-9$, find the equilibrium value (or values) of P, and for values of P less than, between, or greater than these values, determine if P is increasing or decreasing. Illustrate your answer with some possible solution graphs showing all these possibilities.

(6) 7. A tank initially contains 100 l of water, in which 25 g of salt has been dissolved. Pure water enters the tank at a rate of 5 l/min. The solution is kept mixed, and drains from the tank at a rate of 5 l/min. How much salt is in the tank after 20 min?

(Total: 50)