Instructor: Dr. R.A.G. Seely
(March 2017)

Test 2
(version A)

Cal II (S) (Maths 201–NYB)

(Marks)

Justify *all your answers—just having the correct answer is not sufficient.*
Presentation is important, and some credit will be lost for messy or incoherent work.
Pace yourself—a rough guide is to spend not more than $2m$ minutes or so on a question worth m marks.

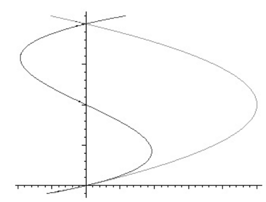
(4×2) 1. Evaluate the following limits:

(a) $\displaystyle\lim_{x\to+\infty}\left(1+\tan\frac{1}{x}\right)^{2x}$ (b) $\displaystyle\lim_{x\to 0}\frac{\arcsin x}{x}$

(c) $\displaystyle\lim_{x\to 0^+}\frac{\cos x}{x}$ (d) $\displaystyle\lim_{x\to+\infty} x(\mathrm{e}^{1/x}-1)$

(3×3) 2. Determine whether these improper integrals converge or diverge: if an integral converges, give the exact value of the integral.

(a) $\displaystyle\int_2^{\infty}\frac{dx}{x(\ln x)^2}$ (b) $\displaystyle\int_0^{3}\frac{dx}{(x-2)^{4/3}}$ (c) $\displaystyle\int_0^{\frac{\pi}{2}}\frac{dx}{\cos x}$



(5) 3. Find the area of the region between the curves $x+3y^2=y^3+2y$ and $x+y^2=2y$ and above the x-axis. (Graphs at right)

(5×2) 4. Find the volumes of the following solids of revolution:

(a) when the region between the curves $y=\sqrt{x+2}$, $y=x$, and $x=0$ is rotated about the y-axis;

(b) when the region between the curves $y=\sqrt{x+2}$, $y=x$, and $y=0$ is rotated about the x-axis. (**Careful:** these are not the same region! One equation has changed.)

(5) 5. Find the length of the curve $y=2\ln(\cos(x/2))$, from $x=0$ to $x=\pi/2$.

(5) 6. Find the general solution of the differential equation: $\cos^2 x\,y'+y\tan x=0,\quad y(0)=1$ Express y as a function of x.

(3) 7. For the differential equation $y'=2y(6-y)-10$, find the equilibrium value (or values) of y, and for values of y less than, between, or greater than these values, determine if y is increasing or decreasing. Illustrate your answer with some possible solution graphs showing all these possibilities.

(5) 8. A certain (fictional!) species of worms is found to have a growth rate where the rate of increase in length is proportional to the difference between its current length and its eventual adult length (in cm). If the eventual length is 6 cm, the initial measured length is 2 cm, and 1 day later its length is 4 cm, then find the differential equation that expresses this relationship, and so find the functional equation $\ell=f(t)$ expressing the length ℓ in terms of time t.

(Total: 50)