Cal II (S) (Maths 201–NYB)

Answers

1. The limits:

   (a) e^2 (b) 1 (c) $+\infty$ (d) 1

2. The improper integrals:

   (a) converges: $\frac{1}{\ln 2}$ (b) diverges: $+\infty$ (c) diverges: $+\infty$

3. $\int_0^2 (2y^2 - y^3)\,dy = \frac{4}{3}$

4. The volumes — there are several ways to do these, *e.g.* (best first):

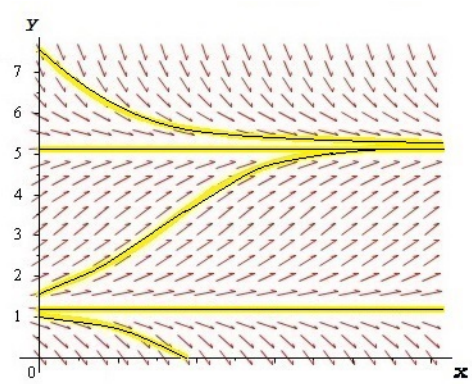
   (a) $V = \int_0^2 2\pi x(\sqrt{x+2} - x)\,dx = \int_0^{\sqrt{2}} \pi y^2\,dy + \int_{\sqrt{2}}^2 \pi(y^2 - (y^2-2)^2)\,dy$
   $= \frac{16\pi}{15}(2\sqrt{2} - 1)$ $(= 6.127118)$

   (b) $V = \int_0^2 2\pi y(y - y^2 + 2)\,dy = \int_{-2}^0 \pi(x+2)\,dx + \int_0^2 \pi(x + 2 - x^2)\,dx$
   $= \frac{16\pi}{3}$ $(= 16.75516)$

5. $2\ln(1 + \sqrt{2})$

6. $y = e^{-\frac{1}{2}\tan^2 x} = e^{\frac{1}{2} - \frac{1}{2}\sec^2 x}$

7. $y' = 0$ if $2y(6-y) - 10 = 0$, *i.e.*
   $y^2 - 6y + 5 = (y-1)(y-5) = 0$, so
   the equilibrium values are at $y_\infty = 1, 5$.
   y increases for $1 < y(0) < 5$,
   and decreases for $y(0) < 1$ or $y(0) > 5$.
   Note that $y = 1$ & $y = 5$ (constant functions) are also solutions.
   Note the graphs also start "exponentially"
   Sample graphs shown (highlighted in yellow if you have a colour printer!).



8. $\frac{d\ell}{dt} = k(6 - \ell)$ so $\ell(t) = 6 - 4\cdot 2^{-t}$ or $\ell(t) = 6 - 4\,e^{-t\ln 2}$