Cal II (S) (Maths 201–NYB)

Answers

1. The integrals:

   (a) $\frac{2}{15}(x^3-1)^{5/2}+\frac{2}{9}(x^3-1)^{3/2}+C$ or $\frac{2}{9}x^3(x^3-1)^{3/2}-\frac{4}{45}(x^3-1)^{5/2}+C$

   (b) $\frac{2}{9}(x^3-1)^{3/2}+C$

   (c) $\frac{1}{2}\ln|2t+\sqrt{4t^2-9}|+C$

   (d) $\frac{1}{2}x^2\arcsin(x)-\frac{1}{4}\arcsin(x)+\frac{1}{4}x\sqrt{1-x^2}\,\Big]_0^{1/2}=\frac{\sqrt{3}}{16}-\frac{\pi}{48}$

   (e) $-2(\sec t)^{-1/2}+C$

   (f) $\frac{1}{25}\tan^5 5\theta+\frac{1}{35}\tan^7 5\theta+C$

   (g) $\frac{1}{2}x^2-2x+\ln|x|+2\ln|x+1|+2/(x+1)+C$

   (h) $2\sqrt{x\ln x-x}+C$

   (i) $\frac{1}{2}t+\frac{1}{4}\sin 2t-\frac{1}{3}\sin^3 2t+\frac{1}{10}\sin^5 2t+C$

   (j) $2\,\mathrm{e}^{1+\sqrt{x}}+C$

   (k) $-\frac{\sqrt{1-x^2}}{x}-\arcsin(x)+C$

   (l) $\frac{1}{10}\mathrm{e}^x\sin 3x-\frac{3}{10}\mathrm{e}^x\cos 3x+C$

   (m) $\frac{1}{4}x^3\mathrm{e}^{4x}-\frac{3}{16}x^2\mathrm{e}^{4x}+\frac{3}{32}x\,\mathrm{e}^{4x}-\frac{3}{128}\mathrm{e}^{4x}+C$

2. $(x\,f'(x)-f(x))\Big|_0^1=1$

3. The derivative is: $-\frac{1}{1+(1+x)^2}=-\frac{1}{2+2x+x^2}$.

4. Simplified: (a) $5/13$ (b) $-\frac{\pi}{4}$ (c) $\frac{\sqrt{1-x^2}}{x}$ This is correct for all x: if x is positive, the angle will be in QI, so the tangent value will be positive also. But if x is negative, the angle will be in QII, where the tangent value will also be negative.

   (d) If $\tan\theta=-\frac{1}{3}$, then $\cos\theta=\pm\frac{3}{\sqrt{10}}$ but $\cos\left(\arctan\left(-\frac{1}{3}\right)\right)=\frac{3}{\sqrt{10}}$. Numerically (in absolute value), these must be equal, but in the former case, θ could be in QII or QIV, and so $\cos\theta$ could be either negative or positive, but in the latter case, the angle $\arctan(-\frac{1}{3})$ must be in QIV, where cos is positive.