Cal II (S) (Maths 201–NYB)

(Marks)

Remember that the use of any calculator is not permitted. Please show all your work, so as to justify your answers. Presentation is important, and some credit will be lost for messy or incoherent work.

Pace yourself—a rough guide is to spend not more than $2m$ minutes or so on a question worth m marks.

(18) 1. Evaluate the following:

(a) $\displaystyle\int_0^{\pi/4} \tan^5(\theta)\, d\theta$ (b) $\displaystyle\int \frac{x^3}{\sqrt{x^2-9}}\, dx$ (c) $\displaystyle\int_0^1 \arctan x\, dx$

(d) $\displaystyle\int x^3 \sin 2x\, dx$ (e) $\displaystyle\int \frac{2x^4-1}{x^2(x-2)}\, dx$ (f) $\displaystyle\int \frac{dx}{\sqrt{3+2x-x^2}}$

(9) 2. Evaluate the following limits:

(a) $\displaystyle\lim_{x\to+\infty} \left(\sqrt{x^2+3x} - x\right)$ (b) $\displaystyle\lim_{x\to 0} \frac{\ln(1-2x)}{\tan x}$ (c) $\displaystyle\lim_{x\to 0} \left(1-\frac{x}{3}\right)^{\frac{4}{x}}$

(8) 3. Determine whether these improper integrals converge or diverge: if an integral converges, give the exact value of the integral.

(a) $\displaystyle\int_0^2 \frac{dx}{(x-1)^2}$ (b) $\displaystyle\int_0^\infty x\, \mathrm{e}^{-x^2}\, dx$

(4) 4. Find the general solution of the differential equation: $x\,y'+1 = y^2$ Express y as a function of x, or x as a function of y (your choice!), using an appropriate constant of integration as necessary.

(3) 5. For the differential equation $P' = P(10-P) - 9$, find the equilibrium value or values of P, and for values of P less than, between, or greater than these values, determine if P is increasing or decreasing. Illustrate your answer with some possible solution graphs showing all these possibilities.

(5) 6. Let $\mathcal{R}$ be the three-sided region in quadrant I between the y-axis and the curves $y = \sin x$ and $y = \cos x$.

(a) Set up the integral necessary to find the area of $\mathcal{R}$.

(b) Set up the integral necessary to find the volume of the solid obtained when $\mathcal{R}$ is rotated about the line $x = -1$.

(c) Set up the integral necessary to find the volume of the solid obtained when $\mathcal{R}$ is rotated about the x-axis.

You do NOT have to evaluate the integrals in question 6!

(3) 7. Find the length of the curve $y = \frac{1}{8}x^2 - \ln x$, from $x = 1$ to $x = 2$.

(Total: 50)