Cal II (S) (Maths 201–NYB)

Answers

1. The integrals:

   (a) $\frac{2}{15}(x^3-1)^{5/2}+\frac{2}{9}(x^3-1)^{3/2}+C$ or $\frac{2}{9}x^3(x^3-1)^{3/2}-\frac{4}{45}(x^3-1)^{5/2}+C$
   (b) $\frac{2}{9}(x^3-1)^{3/2}+C$
   (c) $\frac{1}{2}\ln|2t+\sqrt{4t^2-9}|+C$
   (d) $\frac{1}{2}x^2\arcsin(x)-\frac{1}{4}\arcsin(x)+\frac{1}{4}x\sqrt{1-x^2}\,\Big]_0^{1/2}=\frac{\sqrt{3}}{16}-\frac{\pi}{48}$
   (e) $-2(\sec t)^{-1/2}+C$
   (f) $\frac{1}{25}\tan^5 5\theta+\frac{1}{35}\tan^7 5\theta+C$
   (g) $\frac{1}{2}\left(\operatorname{arcsec}(\sqrt{x})\right)^4\Big]_2^4=\frac{1}{2}\left((\frac{\pi}{3})^4-(\frac{\pi}{4})^4\right)=\frac{175\pi^4}{41472}$
   (h) $2\,\mathrm{e}^{1+\sqrt{x}}+C$
   (i) $\frac{1}{2}t+\frac{1}{4}\sin 2t-\frac{1}{3}\sin^3 2t+\frac{1}{10}\sin^5 2t+C$
   (j) $-\frac{\sqrt{1-x^2}}{x}-\arcsin(x)+C$
   (k) $\frac{1}{10}\mathrm{e}^x\sin 3x-\frac{3}{10}\mathrm{e}^x\cos 3x+C$
   (l) $\frac{1}{4}x^3\mathrm{e}^{4x}-\frac{3}{16}x^2\mathrm{e}^{4x}+\frac{3}{32}x\,\mathrm{e}^{4x}-\frac{3}{128}\mathrm{e}^{4x}+C$

2. $\frac{\pi}{6}+\frac{1}{2\sqrt{3}}$
3. The derivative is: $-\frac{1}{2+2x+x^2}$
4. Simplified: (a) $5/13$ (b) $\frac{\pi}{4}$ (c) $\frac{1}{x}$

   (d) $\tan\theta=\pm\sqrt{8}$ but $\tan\left(\arccos\left(-\frac{1}{3}\right)\right)=-\sqrt{8}$. In the former case, θ could be in QII or QIII, but in the latter case, it must be in QII, where tan is negative.
5. (a) Any x (strictly) between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ will satisfy $\arctan(\tan x)=x$; no x outside that range will, since the **range** of $\arctan x$ is $(-\frac{\pi}{2},\frac{\pi}{2})$.
   (b) Any x will satisfy $\tan(\arctan x)=x$, since the **domain** of $\arctan x$ is $(-\infty,\infty)$.