Cal II (S) (Maths 201–NYB)

In all cases, give a justification of your answers; show sufficient detail so I can see the reasoning you use. The marks you receive will reflect how well you meet this requirement.

1. For each of the following sequences determine whether or not it is convergent. (Justify your answer: if it converges find the limit, otherwise indicate why it diverges.)

   (a) $\{a_k\} = \left\{(-1)^k \dfrac{k+1}{2k-1}\right\}$ (b) $\{b_n\} = \left\{\dfrac{\cos n}{2n-1}\right\}$

2. For the following series calculate (if possible) the sum.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{5+3^n}{4^n}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-1}$ (*Hint:* Factor the denominator!)

3. Classify each of the following series as convergent or divergent. (Briefly justify your conclusions.)

   (a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+1}{2n-1}\right)^n$ (b) $\displaystyle\sum_{k=1}^{\infty} \frac{\ln^2 k}{\sqrt{k}}$ (c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt[3]{\ln(n)}}$

   (d) $\displaystyle\sum_{k=1}^{\infty} \frac{\cos^2 k}{\sqrt[3]{k^4}}$ (e) $\displaystyle\sum_{k=1}^{\infty} \frac{k^3 3^k}{(k+3)!}$

4. Classify each of the following series as absolutely convergent, conditionally convergent or divergent. (Briefly justify your conclusions.)

   (a) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n+1}{n^2+1}$ (b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n e^{-n}$ (c) $\displaystyle\sum_{k=2}^{\infty} (-1)^k \frac{\sqrt{k^3}}{\ln k}$

5. Determine the interval of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n-1}(x-1)^n}{3^n\,(4n+1)}$.

6. For the function $f(x) = \dfrac{1}{x^2}$

   (a) find the first five terms of the Taylor series for $f(x)$ at $x = 1$;

   (b) find the n^{th} term, and express the series in Σ notation.

   (c) What is the radius of convergence for this series?