Instructor: Dr. R.A.G. Seely

Test 3
(version A)

Cal II (S) (Maths 201–NYB)

(Marks)

In all cases, give a justification of your answers; show sufficient detail so I can see the reasoning you use. The marks you receive will reflect how well you meet this requirement.

(6) 1. For each of the following sequences determine whether or not it is convergent. (Justify your answer: if it converges find the limit, otherwise indicate why it diverges.)

(a) $a_n = \dfrac{2^n}{n!}$ (b) $b_n = \left(\dfrac{n}{n+2}\right)^n$

(8) 2. For the following series calculate (if possible) the sum.
(Give your answers in simplified fractional form.)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n - 1}{3^{n-1}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \Big(\arctan(n+1) - \arctan(n)\Big)$

(16) 3. Classify each of the following series as convergent or divergent. (Justify your conclusions.)

(a) $\displaystyle\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(2n)!}{2^n\, n!}$

(c) $\displaystyle\sum_{k=2}^{\infty} \frac{\ln k}{k^2}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{5^{2n}}{n^n}$

(10) 4. Classify each of the following series as absolutely convergent, conditionally convergent or divergent. (Justify your conclusions.)

(a) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n+1}{\sqrt[4]{2n^9 + 6n + 1}}$ (b) $\displaystyle\sum_{k=1}^{\infty} (-1)^{k+1} \frac{2^k}{k}$

(5) 5. Determine the interval of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{4^n\sqrt{n^2+1}}$.

(5) 6. Find the Taylor series for $f(x) = \dfrac{1}{x}$ centered at $x = 1$. What is the interval of convergence for this series?

(Total: 50)