Cal II (S) (Maths 201–NYB)

The Answers

1. (a) Converges to 0 (b) Converges to e^{-2}
2. (a) (GS): $6 - \frac{3}{2} = \frac{9}{2}$ (b) (TS): $\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$
3. (a) (nTT): $\lim n \sin\left(\frac{1}{n}\right) = 1 \neq 0$: $\sum$ diverges
   (b) (RT): $\lim \dfrac{a_{n+1}}{a_n} = \infty > 1$: $\sum$ diverges
   (c) (CT or LCT): $\ln k < \sqrt{k}$ so $\frac{\ln k}{k^2} < \frac{1}{k^{3/2}}$ and $\sum \frac{1}{k^{3/2}}$ C (pS)
   or ($\int$T): f cont, pos, decr; $\int_2^\infty f dx = \frac{1}{2}\ln 2 + \frac{1}{2}$ converges: $\sum$ converges
   (d) ($\sqrt[n]{}$T): $\lim \sqrt[n]{a_n} = \lim \frac{25}{n} = 0 < 1$: $\sum$ converges
4. (a) (LCT) with $\sum \frac{1}{n^{5/4}}$ (C pS): $\lim \frac{a_n}{b_n} = \frac{1}{\sqrt[4]{2}} \neq 0, \neq \infty$ so AC.
   (b) (nTT): $\lim \dfrac{2^k}{k} = \infty \neq 0$: $\sum$ diverges; or by (RT)
5. $-6 \leq x < 2$ ($R = 4$) (Use (AST) and (LCT) at endpoints)
6. $1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 \mp \cdots = \displaystyle\sum_{n=0}^{\infty} (-1)^n (x-1)^n$;
   converges for $0 < x < 2$.