Cal II (Maths 201–NYB)

1. Evaluate the following:

   (a) $\int \sin^4 x\,dx$ (b) $\int \frac{\csc^4 x\,dx}{\sqrt[3]{\cot x}}$ (c) $\int_1^2 x \operatorname{arcsec} x\,dx$

   (d) $\int \frac{(\ln x)^2\,dx}{x}$ (e) $\int e^{-x}\sin 3x\,dx$ (f) $\int \frac{x^3-1}{x^3-2x^2}\,dx$

   (g) $\int_0^4 \frac{x^5}{\sqrt{x^2+9}}\,dx$ (h) $\int \sin(\ln x)\,dx$ (i) $\int (\ln x)^2\,dx$

2. Evaluate the following limits:

   (a) $\lim_{x\to+\infty} \left(1+\tan\frac{1}{x}\right)^{2x}$ (b) $\lim_{x\to 0} \frac{\arcsin x}{x}$

   (c) $\lim_{x\to+\infty} x(e^{1/x}-1)$ (d) $\lim_{x\to 0^+} \frac{\cos x}{x}$

3. Find the particular solution of the following differential equation (satisfying the given initial condition):

   $xy' + y = y^2$ where $y(1) = -1$

4. Determine whether these improper integrals converge or diverge: if an integral converges, give the exact value of the integral.

   (a) $\int_2^\infty \frac{dx}{x(\ln x)^2}$ (b) $\int_0^3 \frac{dx}{(x-2)^{4/3}}$ (c) $\int_0^\infty x\,e^{-x}\,dx$

   (d) $\int_1^\infty \frac{dx}{x^2+x}$ (e) $\int_0^{\frac{\pi}{2}} \frac{dx}{\cos x}$

5. Find the areas of the following regions:

   Note: The graphs are available on my webpage — but calculate the intersection points for yourself; don't rely on the graphs.

   (a) The region between $y = x^3 - 2x^2 + 1$ and $y = 1 - x$.

   (b) The region between the curves $x + 3y^2 = y^3 + 2y$ and $x + y^2 = 2y$ and above the x-axis.

6. (a) Find the volume of the solid generated when the region between the curves $y = \sqrt{x+2}$, $y = x$, and $x = 0$ is rotated about the y-axis.

   (b) Find the volume of the solid generated when the region between the curves $y = \sqrt{x+2}$, $y = x$, and $y = 0$ is rotated about the x-axis.
   (**Careful:** these are not the same region! One equation has changed.)