



Cal II (S) (Maths 201–NYB)

(Marks)

Remember that the use of any calculator is not permitted. Please show all your work, so as to justify your answers. Presentation is important, and some credit will be lost for messy or incoherent work.

(30) 1. Evaluate the following:

(a) $\int_0^1 \frac{t^2 dt}{\sqrt{4-t^2}}$

(b) $\int \frac{x dx}{(25-x^2)^3}$

(c) $\int x^3 \sqrt{x^2+4} dx$

(d) $\int \frac{e^{3x} dx}{1+e^{3x}}$

(e) $\int \sec^4(2x) dx$

(f) $\int x \operatorname{arcsec} x dx$

(g) $\int \frac{e^{3x} dx}{1+e^{6x}}$

(h) $\int \frac{1-\cos t}{1-\sin^2 t} dt$

(i) $\int \frac{dx}{\sqrt{x^2-1}}$

(j) $\int x^3 e^{2x} dx$

(k) $\int \frac{dx}{x+\sqrt{x}}$

(l) $\int \frac{\tan t dt}{\sqrt{\sec t}}$

(m) $\int \sin^3(2\theta) \cos^{11}(2\theta) d\theta$

(n) $\int e^{2x} \sin x dx$

(o) $\int_0^1 \arctan x dx$

(4) 2. Find the derivative $\frac{dy}{dx}$ for the following. Do not simplify your answers.

(a) $y = \frac{\arccos(x)}{\arcsin(x)}$

(b) $y = \tan^{-1}(x) \tan(x)$

(1) 3. (a) Calculate the exact value of $\sin\left(\arccos\left(\frac{12}{13}\right)\right)$.(1) (b) Simplify the expression $\tan(\sec^{-1}(x))$, expressing your answer in a form without any trig or inverse trig.(2) (c) If θ is an angle for which $\tan \theta = -\frac{1}{3}$, then what are all the possible values of $\cos \theta$? What are all the possible values of $\cos\left(\arctan\left(-\frac{1}{3}\right)\right)$?**Briefly** explain the connection between these two answers.(1) 4. (a) Find (if possible) an x for which $\arcsin(\sin x) = x$, and another x for which $\arcsin(\sin x)$ is *not* equal to x .(1) (b) Is it always true that $\sin(\arcsin x) = x$? If so, then *briefly* explain why; and if not, give an example where the equation fails.

(Total: 40)