Cal II (S) (Maths 201–NYB)

(Marks)

Remember that the use of any calculator is not permitted. Please show all your work, so as to justify your answers. Presentation is important, and some credit will be lost for messy or incoherent work.

(36)   1. Evaluate the following:

(a) $\int x^5 \sqrt{x^3 - 1} \, dx$

(b) $\int x^2 \sqrt{x^3 - 1} \, dx$

(c) $\int \frac{dt}{\sqrt{4t^2 - 9}}$

(d) $\int_0^{1/2} x \arcsin x \, dx$

(e) $\int \frac{\tan t}{\sqrt{\sec t}} \, dt$

(f) $\int \sec^4(5\theta) \tan^4(5\theta) \, d\theta$

(g) $\int_2^4 \frac{(\operatorname{arcsec} \sqrt{x})^3}{x\sqrt{x - 1}} \, dx$

(h) $\int \frac{e^{1+\sqrt{x}}}{\sqrt{x}} \, dx$

(i) $\int (\sin^2(t) + \cos^5(2t)) \, dt$

(j) $\int \frac{\sqrt{1 - x^2}}{x^2} \, dx$

(k) $\int e^x \sin 3x \, dx$

(l) $\int x^3 \, e^{4x} \, dx$

(3)   2. What is the slope of the tangent line to the graph of $y = x^2 \arcsin(x)$ at the point where $x = 1/2$?

(3)   3. Find the derivative $\frac{dy}{dx}$ for the function $y = \arctan\left(\frac{1}{1+x}\right)$. Simplify your answer.

(1)   4. (a) Calculate the exact value of $\sin\left(\arctan\left(\frac{5}{12}\right)\right)$.

(1)   (b) Calculate the exact value of $\arctan\left(\sin\left(\frac{\pi}{2}\right)\right)$.

(1)   (c) Simplify the expression $\cos(\sec^{-1}(x))$, expressing your answer in a form without any trig or inverse trig.

(2)   (d) If θ is an angle for which $\cos\theta = -\frac{1}{3}$, then what are all the possible values of $\tan\theta$? What are all the possible values of $\tan\left(\arccos\left(-\frac{1}{3}\right)\right)$? **Briefly** explain the connection between these two answers.

(3)   5. (a) Find (if possible) an x for which $\arctan(\tan x) = x$, and another x for which $\arctan(\tan x)$ is *not* equal to x.

           (b) Is it always true that $\tan(\arctan x) = x$? If so, then *briefly* explain why; and if not, give an example where the equation fails.

(Total: 50)