Calculus II (Maths 201–NYB)

Answers

1. $\displaystyle\int x\ln(x)\,dx$

   Parts: $u=\ln(x), dv=x\,dx$, giving $\displaystyle\int x\ln(x)\,dx=\frac{1}{2}x^2\ln(x)-\frac{1}{4}x^2+C$.

2. $\displaystyle\int\frac{e^{2x}\ dx}{1+e^{4x}}$

   Substitution: $u=e^{2x}, du=2\,e^{2x}\ dx$, and $e^{4x}=u^2$, giving $\int\frac{e^{2x}\ dx}{1+e^{4x}}=\frac{1}{2}\arctan(e^{2x})+C$.

3. $\displaystyle\int x\sqrt{1-x}\,dx$

   Two ways to do this: (1) by back-substitution: $u=1-x, du=-dx$, and $x=1-u$, giving $\displaystyle\int x\sqrt{1-x}\,dx=-\frac{2}{3}(1-x)^{3/2}+\frac{2}{5}(1-x)^{5/2}+C$.
   Also (2) by parts:
   $u=x, dv=(1-x)^{1/2}\,dx$, giving $\displaystyle\int x\sqrt{1-x}\,dx=-\frac{2}{3}x(1-x)^{3/2}-\frac{4}{5}(1-x)^{5/2}+C$.

4. $\displaystyle\int\frac{x^2\,dx}{\sqrt{1-x^2}}$

   Trig Sub: $x=\sin\theta, dx=\cos\theta\,d\theta, \sqrt{1-x^2}=\cos\theta$, giving
   $\displaystyle\int\frac{x^2\,dx}{\sqrt{1-x^2}}=\int\sin^2\theta\,d\theta=\frac{1}{2}\arcsin(x)-\frac{1}{2}x\sqrt{1-x^2}+C$.
   Note you need to use the trig identity $\sin 2\theta=2\sin\theta\cos\theta$.