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Quiz 5
(Version for practice)

Calculus I (Maths 201–NYA)

With Answers

Justify *all your answers—just having the correct answer is not sufficient.*

1. Evaluate the following indefinite integrals:

   (a) $\displaystyle\int \frac{x^4 - 3x^2 + 2x - 7}{3x^3}\,dx$ (b) $\displaystyle\int \left(\frac{\tan x}{\sec x} + \frac{\cot x}{\sin x}\right) dx$ (c) $\displaystyle\int (10^x + x^{10})\,dx$

2. Given $f'(x) = 2x + 2\,\mathrm{e}^x - \sin(x) - \frac{1}{2\sqrt{x}}$, and $f(0) = 5$, find $f(x)$.

3. In each case, express the given limit as a definite integral, starting at the given value of a:

   (a) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{3i}{n}\right)^3 - 6\left(\frac{3i}{n}\right)\right]\frac{3}{n}, \quad a = 0$

   (b) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(2 - \frac{5i}{n}\right)\sqrt{1 + \left(\frac{5i}{n}\right)}\right]\frac{5}{n}, \quad a = 3$

   (c) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left[\tan\left(\frac{2i}{n}\right)\right]\frac{2}{n}, \quad a = -1$

4. In each case, evaluate the definite integral by interpreting it in terms of area.

   (a) $\displaystyle\int_0^1 x\,dx$ (b) $\displaystyle\int_{-2}^1 x\,dx$ (c) $\displaystyle\int_{-1}^2 |x|\,dx$

   (d) $\displaystyle\int_0^2 \sqrt{4 - x^2}\,dx$ (e) $\displaystyle\int_0^2 (x + \sqrt{4 - x^2})\,dx$ (f) $\displaystyle\int_5^5 (x^2 + \sin(x))\,dx$

5. Estimate the definite integral $\displaystyle\int_0^2 x^2\,dx$ using the Riemann sum with 4 terms, with midpoints as sample points.

6. Verify the following: $\displaystyle\int \sec(x)\,dx = \ln|\tan(x) + \sec(x)| + C$ (Hint: differentiation!)
   Note: Be careful to justify the absolute value!

7. What is the derivative $f'(x)$ of the function $f(x) = \displaystyle\int_0^{\mathrm{e}^x} \sqrt{1 + t}\,dt$?

Answers

1. (a) $\dfrac{x^2}{6} - \ln|x| - \dfrac{2}{3x} + \dfrac{7}{6x^2} + C$ (b) $-\cos x - \csc x + C$ (c) $\dfrac{1}{\ln 10}10^x + \dfrac{1}{11}x^{11} + C$

2. $f(x) = x^2 + 2\,\mathrm{e}^x + \cos(x) - \sqrt{x} + 2$

3. (a) $\displaystyle\int_0^3 (x^3 - 6x)\,dx$ (b) $\displaystyle\int_3^8 (5-x)\sqrt{x-2}\,dx$ (c) $\displaystyle\int_{-1}^1 \tan(x+1)\,dx$

4. (a) $1/2$ (b) $-3/2$ (c) $5/2$ (d) π (e) $2+\pi$ (f) 0

5. $\left((\frac{1}{4})^2 + (\frac{3}{4})^2 + (\frac{5}{4})^2 + (\frac{7}{4})^2\right)\frac{1}{2} = \frac{21}{8}$ ($= 2.625$ compared to the actual value $\frac{8}{3} = 2.667$)

6. $\frac{d}{dx}\left(\ln|\tan x + \sec x|\right) = \dfrac{\sec^2 x + \sec x \tan x}{\tan x + \sec x} = \sec x$

   Note: if $f < 0$, $\frac{d}{dx}\left(\ln(-f)\right) = \frac{-f'}{-f} = \frac{f'}{f}$, and if $f > 0$, $\frac{d}{dx}\left(\ln(f)\right) = \frac{f'}{f}$, so $\frac{d}{dx}\left(\ln(|f|)\right) = \frac{f'}{f}$.

7. $\mathrm{e}^x\sqrt{1+\mathrm{e}^x}$