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(Fall 2018)

Assignment 1

Calculus I (Maths 201–NYA)

Derivatives *via* a limit definition

With Answers

Justify *your answers—just having the correct answer is not sufficient.*

1. Using a (correct!) limit definition, find $f'(0)$, where

$$f(x) = \begin{cases} x^3 \sin(\ln(|x|)) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

2. Using a (correct!) limit definition, find $f'(x)$ where $f(x) = (x-3)^{3/2}$.

3. Using a (correct!) limit definition, find $f'(x)$ where

$$f(x) = \frac{2x-1}{|x-3|}$$

(Consider the cases where the fraction is defined.)

Answers

(The answers are "easy", using the derivative formulas — the point here is to use the limit definition of the derivative correctly! I've given hints as to how to do that. Ask if you need a complete solution.)

1. 0

Hint: squeeze theorem! The key thing you need to remember is that $|\sin(A)| \leq 1$. (And see your class notes!)

2. $f'(x) = \frac{3}{2}(x-3)^{1/2}$.

Hint: rationalize the expression you get with the limit definition of $f'(x)$ (and remember that $A^{3/2} = \sqrt{A^3}$). Also, after simplifying $(x+h-3)^3 - (x-3)^3 = h^3 + 3h^2x - 9h^2 + 3hx^2 - 18hx + 27h$.

3. $f'(x) = \begin{cases} \dfrac{5}{(x-3)^2} & \text{if } x < 3 \\ -\dfrac{5}{(x-3)^2} & \text{if } x > 3 \end{cases}$

Hint: $f'(3)$ does not exist, so you only need to worry about the two cases $x < 3$ and $x > 3$. In each case, get the lowest common denominator, and the algebra works out well.