Cal I (S) (Maths 201–NYA)

(Marks)

Justify *all your answers—just having the correct answer is not sufficient.*
Pace yourself—a rough guide is to spend not more than $2m$ minutes or so on a question worth m marks.
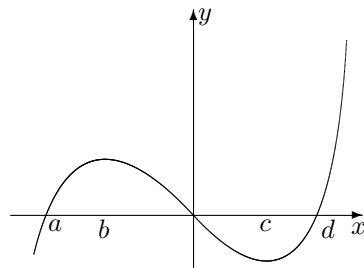
(3)   1. For the following function, find the derivative $f'(x)$ *using a limit definition*. Be sure to state clearly the limit definition of "derivative". Simplify your answer.

$$f(x) = \frac{4}{\sqrt{1 + 2x}}$$

(2)   2. Evaluate the following limit expression, by interpreting it as a derivative (and then use what you know of that derivative).

$$\lim_{h \to 0} \frac{\ln(e + h) - 1}{h}$$

(2)   3. Given the following graph of a function $y = f(x)$,
draw a rough sketch of the graph of the derivative $y' = f'(x)$.

(4×3)   4. For each of the following functions, find the derivative $\dfrac{dy}{dx}$
(using the derivative formulas). You do not need to simplify
your answers.

(a) $y = 16x^4 - \dfrac{4x^7}{21} + \dfrac{2}{3x^4} + \sqrt[3]{e} + \log_7(1 + x^3) - 7^{x^2+1}$

(b) $y = (x^3 - 2x)\sqrt{5x^7 - 6x + \dfrac{1}{x}}$ (c) $y = \dfrac{(5x^2 - 3x + 8)^6}{18} + x^x$ (d) $y = e^{x^3+2x}(5x^4 - \sqrt[3]{x+1})^4$

(5×3)   5. For each of the following functions, find the derivative $\dfrac{dy}{dx}$ (using the derivative formulas). You do not need to simplify your answers. *Hint: You may want to use logarithmic differentiation for some of these.*

(a) $y = \dfrac{(7x^4 - 6x + e^2)^9}{\sqrt[5]{3x^7 - \frac{2}{x} - 2}}$ (b) $y = \sec^3(x^2 - x)\cos^2(5x^4 - 5)$

(c) $y = \dfrac{e^{x^2+1}\sin(2x)}{\sqrt[4]{7x+3}}$ (d) $y = \cot^4\left(\sqrt{x^4 - 7x^6 + 17x}\right)$

(e) $y = (\ln x)^{x^2+1}$

(3)   6. For the equation $x^2y^5 - 3y^2 = \ln(xy^2) - 3$, find the first and second derivatives $\dfrac{dy}{dx}, \dfrac{d^2y}{dx^2}$. What are their values at the point $(1, 1)$?

(3)   7. Find the slope and the equation of the tangent line to the curve $y = \dfrac{2x + 1}{x + 2}$ at the point where $x = 1$.

(3)   8. Find the slope and the equation of the tangent line to the curve $x^3y - y^3 = 3x + y^2$ at the point $(2, 1)$.

(2)   9. Find all x values where the curve $y = 2x^3 + 3x^2 - 36x - 12$ has a horizontal tangent.

(2)   10. Suppose $f(x) = \dfrac{g(x)}{x^2}$, that $g(2) = 3$ and $g'(2) = 5$. What is the value of $f'(2)$?

(3)   11. A ball is given a push down an inclined plane, so that the distance x (in meters) travelled after t seconds is given by $x = 5t + 3t^2$.
What is the velocity v as a function of t?
Find the velocity after 2 s. What is the initial velocity?
How long does it take for the velocity to reach 35 m/s?

(Total: 50)