Cal I (S) (Maths 201–NYA)

Answers

1. $\lim\limits_{h \to 0} \dfrac{1}{h} \left(\dfrac{4}{1 + 2(x + h)} - \dfrac{4}{\sqrt{1 + 2x}} \right) = \lim\limits_{h \to 0} \dfrac{4}{h} \left(\dfrac{\sqrt{1 + 2x} - \sqrt{1 + 2x + 2h}}{\sqrt{1 + 2x}\sqrt{1 + 2x + 2h}} \right) \cdot \left(\dfrac{\sqrt{1 + 2x} + \sqrt{1 + 2x + 2h}}{\sqrt{1 + 2x} + \sqrt{1 + 2x + 2h}} \right)$

   $= \lim\limits_{h \to 0} \dfrac{4}{h} \left(\dfrac{-2h}{\sqrt{1 + 2x}\sqrt{1 + 2x + 2h}\,(\sqrt{1 + 2x} + \sqrt{1 + 2x + 2h}\,)} \right)$

   $= \lim\limits_{h \to 0} 4 \left(\dfrac{-2}{\sqrt{1 + 2x}\sqrt{1 + 2x + 2h}\,(\sqrt{1 + 2x} + \sqrt{1 + 2x + 2h}\,)} \right) = \dfrac{-8}{(1 + 2x) \cdot 2\sqrt{1 + 2x}} = \dfrac{-4}{(1 + 2x)^{3/2}}$

2. This is the derivative of $\ln x$ at $x = \mathrm{e}$, and so $= 1/\mathrm{e}$. (Hint: $\ln(\mathrm{e}) = 1$.)

3. The graph looks like a U-shaped parabola with x-intercepts at b and c:

4. (Note that 4(c) requires you to get the derivative of x^x,
   e.g. with log diff or the equation $a^b = \mathrm{e}^{b \ln a}$.)

   (a) $64x^3 - \frac{28}{21}x^6 - \frac{8}{3}x^{-5} + \frac{3x^2}{(1 + x^3)\ln 7} - 7^{x^2 + 1} \cdot 2x \ln 7$

   (b) $(3x^2 - 2)\sqrt{5x^7 - 6x + \frac{1}{x}} + (x^3 - 2x)\frac{1}{2}(5x^7 - 6x + \frac{1}{x})^{-1/2}(35x^6 - 6 - \frac{1}{x^2})$

   (c) $\frac{1}{3}(5x^2 - 3x + 8)^5(10x - 3) + x^x(1 + \ln x)$

   (d) $\mathrm{e}^{x^3 + 2x}(3x^2 + 2)(5x^4 - \sqrt[3]{x + 1})^4 + \mathrm{e}^{x^3 + 2x}\,4(5x^4 - \sqrt[3]{x + 1})^3(20x^3 - \frac{1}{3}(x + 1)^{-2/3})$

5. (I've done some with log diff, some without. Ask if you want to verify your answer. But note that 5(e) *needs* log diff, unless you use the equation $a^b = \mathrm{e}^{b \ln a}$.)

   (a) $y' = \dfrac{9(7x^4 - 6x + \mathrm{e}^2)^8(28x^3 - 6)\sqrt[5]{3x^7 - \frac{2}{x} - 2} - (7x^4 - 6x + \mathrm{e}^2)^9(\frac{1}{5})(3x^7 - \frac{2}{x} - 2)^{-4/5}(21x^6 + \frac{2}{x^2})}{(3x^7 - \frac{2}{x} - 2)^{2/5}}$

   (b) $y' = 3\sec^2(x^2 - x)\sec(x^2 - x)\tan(x^2 - x)(2x - 1)\cos^2(5x^4 - 5) - \sec^3(x^2 - x)2\cos(5x^4 - 5)\sin(5x^4 - 5)(20x^3)$

   (c) $y' = \left[\dfrac{\mathrm{e}^{x^2 + 1}\sin(2x)}{\sqrt[4]{7x + 3}} \right] \left[2x + \dfrac{2\cos(2x)}{\sin(2x)} - \dfrac{1}{4}\dfrac{7}{7x + 3} \right]$

   (d) $y' = -4\cot^3(\sqrt{x^4 - 7x^6 + 17x})(\csc^2(\sqrt{x^4 - 7x^6 + 17x})\frac{1}{2}(x^4 - 7x^6 + 17x)^{-1/2}(4x^3 - 42x^5 + 17)$

   (e) $y' = (\ln x)^{x^2 + 1}\left[2x \ln(\ln x) + \dfrac{x^2 + 1}{x \ln x} \right]$

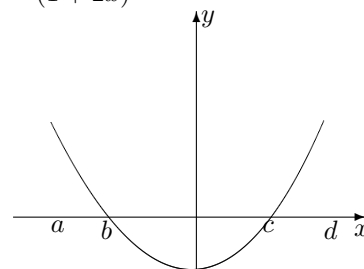6. $y' = \frac{1}{3}$, $y'' = \frac{103}{27}$

7. Slope $= \frac{1}{3}$; Equation: $y = \frac{1}{3}x + \frac{2}{3}$.

8. Slope $= -3$. Equation: $y = -3x + 7$.

9. $x = 2$ or -3

10. $1/2$

11. $v = 5 + 6t$; $v(2) = 17$ m/s, $v_0 = 5$m/s; $t = 5$s

P.S. Let me know if you find (or suspect) any errors!