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Cal I (S) (Maths 201–NYA)

Quiz 1
(version for practice)

With Answers (Let me know if you find any errors!)

1. For each of the following functions, find the derivative $f'(x)$ *using the limit definition.*
   (a) $f(x) = 5x + 7$ (b) $f(x) = \sqrt{x+1}$ (c) $f(x) = 3x^2 + 5$
   (d) $f(x) = \dfrac{3}{x-2}$ (e) $f(x) = \dfrac{1}{\sqrt{x}}$

   Answers:

   (a) $\lim\limits_{h\to 0} \dfrac{(5(x+h)+7) - (5x+7)}{h} = \lim\limits_{h\to 0} \dfrac{5h}{h} = 5$

   (b) $\lim\limits_{h\to 0} \dfrac{\sqrt{x+h+1} - \sqrt{x+1}}{h} = \lim\limits_{h\to 0} \dfrac{(x+h+1)-(x+1)}{h(\sqrt{x+h+1}+\sqrt{x+1})} = \dfrac{1}{2\sqrt{x+1}}$

   (c) $\lim\limits_{h\to 0} \dfrac{(3(x+h)^2+5) - (3x^2+5)}{h} = \lim\limits_{h\to 0} \dfrac{6xh+3h^2}{h} = 6x$

   (d) $\lim\limits_{h\to 0} \dfrac{1}{h}\left(\dfrac{3}{x+h-2} - \dfrac{3}{x-2}\right) = \lim\limits_{h\to 0} \dfrac{-3h}{h(x+h-2)(x-2)} = \dfrac{-3}{(x-2)^2}$

   (e) $\lim\limits_{h\to 0} \dfrac{1}{h}\left(\dfrac{1}{\sqrt{x+h}} - \dfrac{1}{\sqrt{x}}\right) = \lim\limits_{h\to 0} \dfrac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x+h}\sqrt{x}} = \lim\limits_{h\to 0} \dfrac{-h}{h\sqrt{x+h}\sqrt{x}(\sqrt{x}+\sqrt{x+h})} = \dfrac{-1}{2\sqrt{x}^3}$

2. For each of the following functions, find the derivative $f'(x)$ *using the derivative formulas.*
   (a) $f(x) = \sqrt[5]{x^{42}}$ (b) $f(x) = 7x - 3$ (c) $f(x) = 7\sqrt[5]{x} - \dfrac{2}{x^5}$
   (d) $f(x) = x^5 - \dfrac{2}{5x^3} + \sqrt[3]{x^4}$ (e) $f(x) = \dfrac{2x^5 - 7x^3 + 21}{15}$ (f) $f(x) = (6x^{\frac{2}{5}} - 5x^2 + \pi)(2\sqrt{x} + x^2)$
   (g) $y = \dfrac{5x^9 - \frac{1}{x} + 1}{9x^2 - 3x + 5}$ (h) $y = \dfrac{(2x^3-4)^9}{(5x+3x^2+1)^7}$ (i) $y = \left(3x^6 - 4x^2 + 21\right)^{13}(4x-11)^5$

   Answers:

   (a) $\frac{42}{5}x^{37/5}$ (b) 7 (c) $\frac{7}{5}x^{-4/5} + 10x^{-6}$ (d) $5x^4 + \frac{6}{5}x^{-4} + \frac{4}{3}x^{1/3}$ (e) $\frac{2}{3}x^4 - \frac{7}{5}x^2$

   (f) $(\frac{12}{5}x^{-3/5} - 10x)(2\sqrt{x} + x^2) + (6x^{2/5} - 5x^2 + \pi)(x^{-1/2} + 2x)$

   (g) $\dfrac{(45x^8 + x^{-2})(9x^2 - 3x + 5) - (5x^9 - \frac{1}{x} + 1)(18x - 3)}{(9x^2 - 3x + 5)^2}$

   (h) $\dfrac{9(2x^3-4)^8(6x^2)(5x+3x^2+1)^7 - (2x^3-4)^9 7(5x+3x^2+1)^6(5+6x)}{(5x+3x^2+1)^{14}}$

   (i) $13(3x^6 - 4x^2 + 21)^{12}(18x^5 - 8x)(4x-11)^5 + (3x^6 - 4x^2 + 21)^{13}\, 5(4x-11)^4\, 4$

3. Find the slope and the equation of the tangent line to each of the following curves at the given point.
   (a) $y = 5x^3 - 3x^2$ at $x = 1$ (b) $y = \sqrt{x} - 2x + 5$ at $(4, -1)$

   Answers:

   (a) $y = 9x - 7$ (b) $y = -\frac{7}{4}x + 6$

4. Find the equations of the lines tangent to the curve $y = x^3 - 3x^2 - 15x + 7$ which are parallel to the straight line $9x - y + 3 = 0$.

   Answer: Want slope $m = 9$, which is at $x = -2, 4$. At $x = -2$ the equation of the tangent line is $y = 9x + 35$; at $x = 4$ it is $y = 9x - 73$.

5. Find all values of x at which the graph of the following function has a horizontal tangent line: $y = 3x^4 - 10x^3 - 9x^2 + 5$.

   Answer: Horizontal tangents (when $y' = 0$) at $x = 0, -\frac{1}{2}, 3$.

6. Find the values of x for which the lines tangent to the curve $y = x^3 - 3x^2 - 15x + 7$ are normal (*i.e.* at right angles) to the straight line $9x - y + 3 = 0$.

   Answer: Want slope to be $m = -\frac{1}{9}$, which is at $x = 1 \pm \frac{\sqrt{483}}{9}$.