Cal I (S) (Maths 201–NYA)

Some questions to remind you of topics lost in memory(?)
The tiny superscript numbers indicate past exams or textbook problems where you can find the questions (YY-S = Year – Semester, §$x.y$ – z = Text: section – question number). Some brief answers are given here—some others on the exams. Use these to check your work.

1. Limits:
   (a) $\lim_{h\to 0} \dfrac{\sec(h)-1}{h}$ (b)$^{(12\text{-}3)}$ $\lim_{x\to\infty}(e^x - e^{2x})$ (c)$^{(11\text{-}3)}$ $\lim_{\theta\to 0} \dfrac{\theta^2-\theta}{\tan(4\theta)}$

   [Ans: $0, -\infty, -\frac{1}{4}$]

2. Derivatives: Find $\frac{dy}{dx}$:
   (a)$^{(11\text{-}1)}$ $y = \tan^3(x)\csc(10x-1)$ (b)$^{(12\text{-}3)}$ $y = 5^{\cot x} + \sec(4x^2) - 2\,\mathrm{e}^{\pi+1}$
   (c)$^{(14\text{-}3)}$ $y = \dfrac{(\mathrm{e}^x + x^4)^3}{\sec(5x)+1}$ (d)$^{(12\text{-}1)}$ $y = (x^2+1)^{\csc(x)}$
   (e)$^{(16\text{-}1)}$ $y = \tan(x^9 + x^x)$ (f)$^{(14\text{-}3)}$ $x^y = y^{\sin x}$

   [Ans: see exam solutions]

3. $^{(13\text{-}3)}$ Prove that the equation $\mathrm{e}^x = 2 - x$ has exactly one real root.
   [Ans: Rolle's and Intermediate Value theorems]

4. $^{(\S 3.9\text{-}47)}$ A plane flying at 300 km/h passes over a station at an altitude of 1km, and climbs at an angle of 30°. At what rate is the distance between the plane and the station increasing a minute later? (Hint: Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$)
   [Ans: approx 296.35]

5. $^{(98\text{-}3)}$ A right triangle is formed in the first quadrant by the x and y axes and a line through the point $(7,5)$. What should the vertices be in order that the triangle have as small an area as possible?
   [Ans: $(0,10),(14,0)$]

6. $^{(14\text{-}1)}$ Given $f(x) = \dfrac{6-2\,\mathrm{e}^x}{\mathrm{e}^x+1}$ $f'(x) = -\dfrac{8\,\mathrm{e}^x}{(\mathrm{e}^x+1)^2}$ $f''(x) = \dfrac{8\,\mathrm{e}^x(\mathrm{e}^x-1)}{(\mathrm{e}^x+1)^3}$
   Draw a sketch of the graph of the function, identifying all intercepts, aymptotes, local extrema, and inflection points. Specify the domain of the function, and the intervals where the graph is increasing, decreasing, concave up, and concave down.
   [Ans: see exam solutions]

7. $^{(14\text{-}1)}$ Evaluate $\displaystyle\int_{-2}^{2} \left(|x| + \sqrt{4-x^2}\right) dx$ by interpreting it in terms of area.
   [Ans: see exam solutions]

There are many other types of problems you must be prepared for—this list is nowhere near total! Look at old exams, your past class tests and quizzes (and their practice tests and quizzes), and some homework questions. Try at least one old exam in real time (give yourself no more than 3 hours) to see how you cope, and then focus on the areas you have difficulty with.