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Test 2

(short practice version)

Cal I (S) (Maths 201–NYA)

(Marks)

Justify *all your answers—just having the correct answer is not sufficient.*

Pace yourself—a rough guide is to spend not more than 2m minutes or so on a question worth m marks.

(5×3) 1. Calculate the following limits (if they exist). If a limit does not exist, say so, and if appropriate one-sided limits exist instead, state them explicitly. If any limits are infinite, state this explicitly as well.

(a) $\lim_{x\to-\infty} \dfrac{4+5x}{\sqrt{5+4x^2}}$ (b) $\lim_{x\to-1} \dfrac{2x^2-x-3}{x^2+3x+2}$ (c) $\lim_{x\to-2} \dfrac{2x^2-x-3}{x^2+3x+2}$

(d) $\lim_{x\to5} \sqrt[3]{\dfrac{3x+1}{2x^2+4}}$ (e) $\lim_{x\to+\infty} \dfrac{3x^3-2x^2+5}{2x^3-5x-7}$

(3) 2. For the function $f(x) = \begin{cases} \dfrac{2x^2-3x-2}{2x^2-5x+2} & \text{if } x<2 \\ 3/5 & \text{if } x\geq 2 \end{cases}$

find all the values of x for which the function is discontinuous. For each, specify if the discontinuity is removable or not. (Justify)

(3) 3. For the function $h(t) = \begin{cases} a\,t^2-1 & \text{if } t<3 \\ 5-\sqrt{3\,t} & \text{if } t\geq 3 \end{cases}$

find all values of a that make $h(t)$ continuous at $t=3$.

(3) 4. For the function $f(x) = \dfrac{3x^2-4x+1}{9x^2-1}$

find all the values of x for which the function is discontinuous. For each discontinuity, specify what sort of discontinuity it is. If the discontinuity is removable, redefine the function at that point to remove the discontinuity.

(3) 5. Find the derivative of $y=(x^2+1)^{\cos x}$. You do **not** need to simplify your answer. (Use logarithmic differentiation if appropriate.)

(4) 6. Find the second derivative of $y=\dfrac{5x-3}{2x+7}$; simplify your answer.

(3) 7. Draw the graphs of functions which have the indicated properties (if a property is impossible, say so, and explain why):

(a) The function is continuous everywhere, but not differentiable at $x=0$.

(b) The function is continuous everywhere except at $x=0$, but at $x=0$ the discontinuity is removable.

(c) The function is differentiable everywhere, but is discontinuous at $x=0$.

(4) 8. A ladder 13.0 m long is leaning against a wall. The base of the ladder is sliding away from the wall at a rate of 1.75 m/s. How fast is the top of the ladder sliding down the wall at the instant when its base is 5.0 m away from the wall? (Give your answer accurate to 3 significant figures.)

(4) 9. Two cars start moving away from the same point at the same time: one travels south at 60 km/hr, the other travels east at 25 km/hr. At what rate is the distance between them increasing an hour and a half later?

(4) 10. Find the values of x where the absolute (or global) minimum and maximum values of the function $f(x)=x\sqrt{1+x}$ occur on the interval $[-1,1]$ (*i.e.* $-1\leq x\leq 1$).

(4) 11. Find the values of x where the (absolute) minimum and maximum values of the function $f(x)=x-\frac{1}{x^2}$ occur. If the function has no absolute minimum (or no absolute maximum), say so and justify your answer.

(Total: 50)