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Test 3
(version A)

Cal I (S) (Maths 201–NYA)

(Marks)

(3) 1. For $y = x^2\sqrt{1+2x}$ calculate y' and y'' and simplify them both completely, so that your answers are simple fractions.

(3) 2. Sketch the graph of a function $f(x)$ having all of the following properties.

(a) f has horizontal asymptotes at $y = -2$ and $y = 2$.
(b) f has a vertical asymptote at $x = 1$.
(c) f is continuous but not differentiable at $x = -2$.
(d) f has a removable discontinuity at $x = 0$.
(e) f is increasing on the intervals $-\infty < x < -2$, $-1 < x < 0$, $0 < x < 1$, $1 < x < 2$.
(f) f is decreasing on the intervals $-2 < x < -1$, $2 < x < +\infty$.
(g) f is concave up on the intervals $-\infty < x < 1$, $3 < x < +\infty$.
(h) f is concave down on the interval $1 < x < 3$.

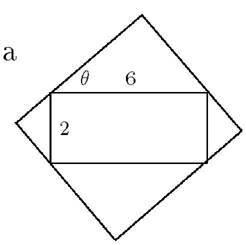
(7) 3. For the following function, graph the function, identifying all intercepts, aymptotes, local extrema, and inflection points. Specify intervals where the graph is increasing, decreasing, concave up, and concave down. Show all your work. To help you on your way, the first and second derivatives are already calculated for you.

$$y = \frac{x}{(x+1)^2} \qquad y' = \frac{1-x}{(x+1)^3} \qquad y'' = \frac{2x-4}{(x+1)^4}$$

(2) 4. Consider a cubic function $y = x^3 + ax^2 + bx$ which goes through the point $(1,1)$ and has a point of inflection there. Find the values of a and b.

(6) 5. Find the values of x where the minimum and maximum values of the function $f(x) = x - \sqrt{1+x}$ occur on the interval $[-1, 1]$.

(5) 6. Find the maximum area of a rectangle that can be circumscribed about a (fixed) rectangle of length 6 and width 2. (Hint: consider the angle θ.)



(4×4) 7. Evaluate the following:

(a) $\displaystyle\int \frac{3 - 5x^3 + 2\sqrt[3]{x}}{x^4}\,dx$

(b) $\displaystyle\int_1^2 x(x-1)^2\,dx$

(c) $\displaystyle\int_0^1 (\sqrt{t} - e^t)\,dt$

(d) $\displaystyle\int \frac{1-\sin t}{1-\sin^2 t}\,dt$ (Hint: Use some trig identities, including Pythagoras)

(3) 8. What is the derivative $f'(x)$ of the function $f(x) = \displaystyle\int_0^{\sin x} \frac{1}{1+t}\,dt$?

(5) 9. Find the area under the curve $y = \sqrt[3]{x} - x$, above the x-axis, and to the right of the y-axis. (The graph is shown at right.)