



Cal I (S) (Maths 201-NYA)

1. For each of the following functions, calculate the derivative $\frac{dy}{dx}$. Do not simplify your answers. (Use logarithmic differentiation where appropriate.)
- (a) $y = \sec(2x^7 - 7x^2)$ (b) $y = \csc(x^5 + 2) \tan^2(x^3 - 8)$ (c) $y = e^{2x^3+1} - 3 \ln(x^2 + 1)$
 (d) $y = \frac{\sqrt{8 - 3x^5}}{\ln(7x^4 - 5x)}$ (e) $y = e^{x^3+1} \sin(x^2 - 1)$ (f) $y = \ln^4(\cot(x^5 - 7x^4 + 17))$
 (g) $y = (\ln x)^{x^2+1}$ (h) $y = e^{x^3+1} \ln(3x^5 - \sec x)$ (i) $y = \frac{x(4x^3 - 1)^{31} \sqrt[5]{x^7 - 2x^4}}{\sqrt[7]{3x - 5} (x^6 - 5x^3)^{17}}$
 (j) $y = \cos(x^5 + 2) \tan(x^3 - 8)$ (k) $y = 5^{2x} - 3 \log_7(x^2 + 1)$ (l) $y = \frac{\sqrt{8 - 3x^5}}{(7x^4 - 5x)^{32}}$
 (m) $y = (\sin x)^{x^2+1}$ (n) $y = \log_7(x^2 + \ln x)$ (o) $y = \frac{\ln(4x^3 - 1)}{\sqrt[7]{3x - 5}}$
 (p) $y = \cot^4(\sqrt[3]{5x^3 - 2x^5})$ (q) $y = (x + 1)^{\cos x}$ (r) $y = \ln\left(\frac{2^x \cos^3(x^4 - 10)}{\sqrt[3]{3x^4 + 2x - 5}}\right)$
 (s) $y = (3x^7 - 5x + 1)^{(x^3 - x)}$ (t) $y = e^{(1 - x^2)} + \tan(2x + 5)$ (u) $y = \left(\frac{(2x^7 - 2 \sec x + e)^{24} \sin^2(x^4 - 2)}{(2x^5 - 3x^2 + 9)^6 \sqrt[9]{\ln x - 5x^2 + 7}}\right)^{12}$
 (v) $y = \ln\left(\frac{\sqrt[3]{3x^4 + 2x - 5}}{2^x \cos^3(x^4 - 10)}\right)$ (w) $y = \log_5(x^2 + \ln x)$ (x) $y = e^{2 \cos x} - 3 \ln(x^4 + 10x - 2)$
- 2.
- (a) $y = \csc^8(\sqrt{3x^4 - 25x^2})$ (b) $y = e^{x^2-1} \ln(x^3 + 1)$ (c) $y = e^{(x^2 + 3x^5)} + \tan(2x + 5)$
 (d) $y = 7^{2x} - 3 \log_5(x^4 + 1)$ (e) $y = \tan^6(\ln(3x^8 - e^{x^3-8}))$ (f) $y = \log_5\left(\frac{\csc^3(3x^2 - 10x)}{5^x \sqrt{x^3 - 5}}\right)$
 (g) $y = e^{x^3+1} \ln(x^2 - 1)$ (h) $y = (7x^3 - 5x + 1)^{(x^7 - x)}$ (i) $y = \frac{(4x^9 - 9x^4 + 49)^8 \sqrt[7]{6x^3 - 5x + 7}}{(3x^5 - 2x^3 + \pi)^{24} \sin^4(x^2 + 1)}$
3. For each of the following, find the second derivative $\frac{d^2y}{dx^2}$. Simplify as appropriate.
- (a) $y = \ln(\sin x)$ (b) $x \tan(x^2 + 1)$ (c) $y = x \ln(x^2 + 1)$
 (d) $y = \csc(x^2)$ (e) $y = \ln(x + \tan x)$ (f) $y = \frac{x - 4}{3x + 5}$
4. For each of the following equations, find the first and second derivatives $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$:
- (a) $x^2y^5 - 3y^2 = \ln(xy^2) - 3$ (b) $y^3 - x^5 = \ln(x^4y^5) + 23$ (c) $\sin^3 y - x^5 = \ln(3x^4y^5) + \pi^2$
5. Find the slope and the equation of the tangent line to the following curves at the given point:
- (a) $y = \frac{2x - 1}{x^2 - 1}$ at $x = 2$ (b) $y = \frac{x^2 - 1}{2x - 1}$ at $x = 2$ (c) $xy^3 - x^2 = 3y + x^3$ at $(1, 2)$
 (d) $y^3 - 2x^2y + 1 = \sin(x - y^2)$ at $(1, 1)$ (e) $y^2 - 2x^2 - 1 = e^{2x - y}$ at $(1, 2)$
6. At a certain moment each edge of a cube is 10 cm long, and the surface area is increasing at a rate of 2.25 cm²/sec. How fast is the volume of the cube increasing?
7. There is a picture of a square on a computer screen; the length of each side of the square is increasing at the rate of 0.25 cm/sec. Find the rate at which the area of the square is increasing when each side is 7.50 cm long.
8. An oil spill spreads in a circle whose radius increases at a constant rate of 10 m/sec. How rapidly is the area of the spill increasing when the radius is 120 m? (Hint: the area of a circle is $A = \pi r^2$.)

9. A conical water tank is being drained at a constant rate. The tank is 15 m high and 8m in diameter (at its top). The water level is falling at a rate of 75 cm/min when the level is 6 m. Find the rate at which the tank is being emptied. (Hint: $V = \frac{1}{3}\pi r^2 h$)
10. Another conical water tank is being filled at a rate of 20 ft³/hr. This tank is 7 ft high with a 6 ft radius at the top. What is the rate at which the water level is rising when the water surface is 5 ft in diameter?
11. In an engine cylinder, the relationship between the pressure p (in kPa) and volume v (in cm³) of the gas vapor is given by the equation $pv^2 = K$, for a constant K . At a certain time, the pressure and volume are determined to be $p = 4000$ kPa, $v = 80$ cm³, and the volume is increasing at a rate of 800 cm³/sec. At that time, what is the rate of change (with respect to time) of the pressure?
12. An circular oil spill is increasing in size, so that the area is increasing at a rate of 50 cm²/min. What is the rate of change (with respect to time) of the radius at the moment when the radius is 15 cm?
13. Let θ (in radians) be an acute angle in a right angled triangle, and let x, y be the lengths of the sides adjacent and opposite θ respectively. Suppose that x and y vary with time t . (a) What is $\frac{d\theta}{dt}$ in terms of $\frac{dx}{dt}$ and $\frac{dy}{dt}$? (Hint?: think of $\tan \theta$.) (b) At a certain instant, $x = 2$ cm and is increasing at 1 cm/s, while $y = 2$ cm and is decreasing at $\frac{1}{4}$ cm/s. How fast is θ changing at that instant.
14. A ladder 5 m long is leaning against a wall. The base of the ladder is sliding away from the wall at a rate of 1 m/s. How fast is the top of the ladder sliding down the wall at the instant when its base is 3 m away from the wall?
15. An inflated spherical balloon is punctured and gradually loses air at a constant rate of 10 cm³/min. If the original diameter was 20 cm, find the rate at which the diameter is decreasing when it is: (a) 15 cm in diameter; (b) 10 cm in diameter; (c) half its original volume.
16. Calculate the following limits (if they exist). If a limit does not exist, say so, and if appropriate one-sided limits exist instead, state them explicitly. If any limits are infinite, state this explicitly as well.

$$\begin{array}{lll} \text{(a)} \lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 + 2x - 3} & \text{(b)} \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} & \text{(c)} \lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x - 3} \\ \text{(d)} \lim_{x \rightarrow 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x} & \text{(e)} \lim_{x \rightarrow 7} \frac{2x - 1}{x - 7} & \text{(f)} \lim_{x \rightarrow 7} \frac{x - 7}{2x - 1} \end{array}$$

17.

$$\begin{array}{lll} \text{(a)} \lim_{x \rightarrow +\infty} \frac{4 - 2x^5 + 3x^3}{5 + 7x^5 - 2x^3} & \text{(b)} \lim_{x \rightarrow -\infty} \frac{4 + 5x}{\sqrt{5 + 4x^2}} & \text{(c)} \lim_{x \rightarrow 1} \frac{\frac{3}{x+2} - 1}{x - 1} \\ \text{(d)} \lim_{x \rightarrow 1} \frac{x^2 + 3x - 4}{2x^2 + x - 3} & \text{(e)} \lim_{x \rightarrow -1} \frac{x^2 - 2x + 3}{2x^2 - 9x - 9} & \text{(f)} \lim_{x \rightarrow 1} \frac{\sqrt[3]{1+x} - \sqrt[3]{1-x}}{x} \\ \text{(g)} \lim_{x \rightarrow 3} \frac{x^2 + 1}{\sqrt{3-x}} & \text{(h)} \lim_{x \rightarrow \infty} \frac{x^2 + 3x - 4}{2x^2 + x^3 - 3} & \text{(i)} \lim_{x \rightarrow 0} \frac{x^2 - x - 2}{x^2 - 4x + 4} \\ \text{(j)} \lim_{x \rightarrow +\infty} \frac{3x^2 - 2x + 5}{2x^2 - 5x - 7} & \text{(k)} \lim_{x \rightarrow -\infty} \frac{\sqrt{4x^6 + 4x^3 - 1}}{7x^3 - x^2 + 9} & \text{(l)} \lim_{x \rightarrow 2} \frac{\frac{2}{x-1} - x}{x - 2} \\ \text{(m)} \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 4x + 4} & \text{(n)} \lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{2x^2 - 9x + 9} & \text{(o)} \lim_{x \rightarrow 1} \frac{x^2 + 3x - 1}{2x^2 + x - 3} \\ \text{(p)} \lim_{x \rightarrow 3^-} \frac{|5x - 15|}{x - 3} & \text{(q)} \lim_{x \rightarrow +\infty} \frac{4 - x^2 + 3x^3}{7 + x^2 - 5x^3} & \text{(r)} \lim_{x \rightarrow -\infty} \frac{\sqrt{2+9x^2}}{9 + 2x} \\ \text{(s)} \lim_{x \rightarrow 1} \frac{\frac{2}{x+1} - 1}{x - 1} & \text{(t)} \lim_{x \rightarrow -1} \frac{x^2 + 3x - 1}{2x^2 + x - 3} & \text{(u)} \lim_{x \rightarrow 3} \frac{x^2 - 2x + 3}{2x^2 - 9x + 9} \\ \text{(v)} \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} & \text{(w)} \lim_{x \rightarrow 1} \frac{1 - \sqrt{2-x}}{x - 1} & \text{(x)} \lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{2x^2 - x - 3} \end{array}$$

18. For the function $f(x) = \begin{cases} 3kx + 1 & \text{if } x < 2 \\ x^2 - k & \text{if } x \geq 2 \end{cases}$ find a value of k which makes the function continuous at $x = 2$.

19. Let $f(x) = \begin{cases} ax^2 + b & \text{if } x < 1 \\ x - a & \text{if } x \geq 1 \end{cases}$. Find values of a and b that make $f(x)$ continuous at $x = 1$.

Check with me if the following topics are on Test 2 or Test 3.

31. For the following functions, determine at what points (x, y) the function has relative extrema and points of inflection? On what intervals is the function increasing? On what intervals is the function decreasing? Sketch a graph of the function; make sure your graph clearly illustrates all these features.

(a) $y = x + \frac{4}{x^2}$

(b) $y = 5x^6 - 3x^5 + 10$

32. For the function $y = x^4 - 4x^2 + 3$, find the intervals where the function is: increasing, decreasing, concave up, concave down. Find the relative maximums and minimums. (A table of this data will suffice.) Draw a rough sketch of the curve.

33. For the following function find all intercepts, asymptotes, relative extrema, and points of inflection. (You should tabulate the data.) Use this information to sketch a graph of the function. (Clearly label the relevant points.)

(a) $f(x) = x^4 - \frac{8}{3}x^3 + 5$

(b) $f(x) = \frac{x}{(x+1)^2}$

(c) $f(x) = \frac{x}{(x+2)^2}$