1 Various

1. Find all the real values of $x$ for which the derivative of the function defined by $k(x) = \frac{x^2}{e^x}$ is zero.

2. Find all the critical numbers for $y = \frac{e^{x^2}}{x^2}$.

3. Find the point on the graph of $y = \sqrt{x}$ which is closest to the point $(4, 0)$.

4. Consider $f(x) = x^2 + 2x$ on $[0, 2]$. Find at least one value of $x$ in the interval $(0, 2)$ for which the slope of the tangent to $y = f(x)$ is parallel to the line segment joining the points $(0, 0)$ and $(2, 8)$.

5. Find all critical numbers of the first derivative of $y = x^5e^{-3x}$.

6. Given $y = x\sqrt{8-x^2}$. Find all values of $x$ such that $\frac{dy}{dx} = 0$.

7. If $y = (3x - 4)(2x - 1)^2$ find all values of $x$ for which $\frac{dy}{dx} = 0$.

8. Given $f(x) = \frac{x^2 + 2}{x^2 - 4}$
   (a) Find $f'(x)$ and simplify.
   (b) Find all critical values of $f$.
   (c) Find all vertical and horizontal asymptotes.
   (d) Find any absolute extrema on the interval $[-1, 2]$.

9. Find $f(x)$ given (i) $f(1) = e + 2$, (ii) $f'(1) = e + 2$, and (iii) $f''(x) = e^x - \frac{1}{x^2}$.

10. Find all values of $x$ such that $f'(x) = 0$ if $f(x) = (3 - x)^3(2x + 1)^2$.

11. The position of a particle at time $t$ is given by $s = \frac{1}{t} \sin 3t$. Find its velocity $\frac{ds}{dt}$, and its acceleration $\frac{d^2s}{dt^2}$ when $t = \frac{\pi}{6}$.

12. Find $f'$ and simplify. State all critical numbers for $f$.
   (a) $f(x) = x^4 \ln x$
   (b) $f(x) = \frac{x^{3/2}}{x^2 + 1}$

13. Given the function $f(x) = \frac{x^2}{e^x}$, specify the interval(s) over which $f(x)$ is increasing.

14. Given $f(x) = 2\sin x + \sin 2x$, Determine if the function concave up, concave down, or neither when $x = \frac{\pi}{2}$. Give your reason.

15. Give the equations of all asymptotes of the function $y = \frac{x^2 + 15x - 16}{1 - x^2}$.

\textbf{Answers:}

1. $x = 0, 2$
2. $x = 0, \pm 1$
3. $(\frac{\pi}{2}, \sqrt{\frac{\pi}{2}})$
4. 1.
5. 0, $\frac{3}{2}$
6. $\pm 2$.
7. $\frac{1}{2}$ and $\frac{19}{18}$
8. (a) $f''(x) = \frac{-12x}{(x^2 - 4)^2}$
   (b) $x = 0$.
   (c) Vertical asymptotes: $x = \pm 2$
   Horizontal asymptote: $y = 1$
   (d) Absolute maximum at $(0, -\frac{1}{2})$.
9. $f(x) = e^x + \ln |x| + x + 1$
10. $x = 3, -\frac{1}{2}, \frac{3}{2}$
11. $\frac{ds}{dt}\bigg|_{t=\frac{\pi}{6}} = 0, \frac{d^2s}{dt^2}\bigg|_{t=\frac{\pi}{6}} = -\frac{9}{2}$
12. (a) $f'(x) = x^3(4 \ln x + 1)$
   Critical numbers $x = 0, e^{-1/4}$
   (b) $\frac{1 - 4x}{3(2x + 1)^2x^{2/3}}$
   Critical numbers $x = \frac{1}{4}$
13. $[0, 2]$.
14. Concave down since $f''(\frac{\pi}{2}) = -2 < 0$.
15. Vertical asymptotes: $x = -1$ only
   Horizontal asymptote: $y = -1$