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Calculus II – Final Exam Problems
Limits

Evaluate each limit.

1. $\lim_{x\to 0} \dfrac{x^2}{\ln\cos x}$
2. $\lim_{x\to 0} \dfrac{e^x - e^{-x} - 2x}{x - \sin x}$
3. $\lim_{x\to 0^+} (\tan x)\ln x$
4. $\lim_{x\to 0} \dfrac{\sin x - x\cos x}{x - \sin x}$
5. $\lim_{x\to 1^+} \left(\dfrac{1}{\ln x} - \dfrac{1}{x-1}\right)$
6. $\lim_{x\to 0} \dfrac{\tan x - \sin x}{x^3}$
7. $\lim_{x\to \frac{\pi}{2}^-} (2x\tan x - \pi\sec x)$
8. $\lim_{x\to 0^+} \left(\dfrac{1}{x-1} - \dfrac{2}{x^2-1}\right)$
9. $\lim_{x\to 0^+} \dfrac{e^{-1/x}}{x}$
10. $\lim_{x\to 0} \dfrac{\sin x - x\cos x}{x^2\sin x}$
11. $\lim_{x\to 0^+} \dfrac{x - \arctan x}{x^3}$
12. $\lim_{x\to\infty} \left(1 - \dfrac{1}{x}\right)^x$
13. $\lim_{x\to 0} \left(\dfrac{1}{x} - \dfrac{\ln|x+1|}{x^2}\right)$
14. $\lim_{x\to\infty} \left(1 - \sin\dfrac{1}{x}\right)^{2x}$
15. $\lim_{x\to\infty} x^2 e^{-x}$
16. $\lim_{x\to 0^+} \left(\dfrac{1}{\sin x} - \dfrac{1}{x}\right)$
17. $\lim_{x\to 2} \left(\dfrac{1}{\ln(x-1)} - \dfrac{1}{x-2}\right)$
18. $\lim_{x\to 0} \dfrac{\tan x - x}{x - \sin x}$
19. $\lim_{x\to 0^+} (1+2x)^{1/x^2}$
20. $\lim_{x\to 0^+} \dfrac{1 - \sin 2x}{x}$
21. $\lim_{x\to\infty} e^{-x}\ln x$
22. $\lim_{x\to 0^+} (1-2x)^{1/x}$
23. $\lim_{x\to 0^-} \dfrac{\cos(\frac{\pi}{2} - x)}{\arctan x}$
24. $\lim_{x\to 0^+} (1-2x)^{\cot x}$
25. $\lim_{x\to \frac{\pi}{2}^-} \dfrac{\sec x + 5}{3\tan x}$
26. $\lim_{x\to\infty} x\ln\left(1 + \dfrac{a}{x}\right)$
27. $\lim_{x\to\infty} x - \sqrt{x^2 - 10}$
28. $\lim_{x\to 0^+} (e^x + x)^{1/x}$
29. $\lim_{x\to\infty} x\sin\dfrac{1}{x}$
30. $\lim_{x\to \frac{\pi}{2}} (\sec x - \tan x)$
31. $\lim_{x\to 0^+} (x^x)^x$
32. $\lim_{x\to 0} \dfrac{e^x - e^{-x}}{\sin x}$
33. $\lim_{x\to 0^+} \left(\dfrac{1}{x} + \ln x\right)$
34. $\lim_{x\to\infty} \left(1 + \dfrac{1}{3x}\right)^x$
35. $\lim_{x\to 0} (1-2x)^{2/x}$
36. $\lim_{x\to 0^+} \dfrac{e^{2x}}{x\cos x}$
37. $\lim_{x\to 0} \dfrac{x - \sin x}{x - \tan x}$
38. $\lim_{x\to 0} (1 + \tan x)^{1/x}$
39. $\lim_{x\to 1} \left(\dfrac{1}{\ln x} - \dfrac{x}{\ln x}\right)$
40. $\lim_{x\to 0^+} \dfrac{\ln x}{\csc x}$
41. $\lim_{x\to 0^+} \left(\dfrac{2}{\sin^2 x} - \dfrac{1}{1-\cos x}\right)$
42. $\lim_{x\to\infty} \left(1 - \dfrac{3}{x}\right)^x$
43. $\lim_{x\to\infty} \dfrac{x^3}{e^x}$
44. $\lim_{x\to 0} (1 + \sin x)^{1/3x}$
45. $\lim_{x\to 0} \dfrac{xe^{2x}}{4\tan x}$
46. $\lim_{x\to 0^+} \left(\cot x - \dfrac{1}{x}\right)$
47. $\lim_{x\to 0^+} \left(\dfrac{1}{x}\right)^{\sin x}$
48. $\lim_{x\to\infty} \left(\dfrac{x+2}{x}\right)^x$
49. $\lim_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{e^x - 1}\right)$
50. $\lim_{x\to 1^+} \left(\dfrac{1}{\ln x} + \dfrac{1}{x-1}\right)$
51. $\lim_{x\to 2\pi} \left(3 - \dfrac{x}{\pi}\right)^{\csc x}$
52. $\lim_{x\to -\frac{\pi}{2}} \left(\dfrac{\pi}{2} + x\right)\sec x$
53. $\lim_{x\to 1} \dfrac{1 - x + \ln x}{1 + \cos\pi x}$
54. $\lim_{x\to 0} (1+3x)^{\csc x}$

55. $\lim_{x\to\frac{\pi}{2}^-} (\sec x - \tan x)$

56. $\lim_{x\to\frac{\pi}{2}^-} (\cos x)\ln\cos x$

57. $\lim_{x\to 0} \dfrac{e^{2x} - e^{-2x} - 4x}{x^2}$

58. $\lim_{x\to\infty} (1+8x^2)^{1/x^2}$

59. $\lim_{x\to 0} \left[\dfrac{1}{\ln(1+x)} - \dfrac{1}{x}\right]$

60. $\lim_{x\to 0} \dfrac{1-\cos 2x}{x^2}$

61. $\lim_{x\to 0} (1+x)^{1/x}$

62. $\lim_{x\to\frac{\pi}{2}} \dfrac{\cot x - \cos x}{x^2}$

63. $\lim_{x\to 0^+} (1+2x)^{1/x}$

64. $\lim_{x\to 0^+} \dfrac{\ln\tan 2x}{\ln\sin 3x}$

65. $\lim_{x\to 0} (e^x + x)^{1/x}$

66. $\lim_{x\to\frac{\pi}{2}^-} \cos 3x \sec 7x$

67. $\lim_{x\to\infty} \left(1+\dfrac{3}{x}\right)^{2x}$

68. $\lim_{x\to 0} \dfrac{e^x - 1}{\sin x}$

69. $\lim_{x\to 0^+} \dfrac{\ln x}{\csc x}$

70. $\lim_{x\to 0^+} x^{1/\ln x}$

71. $\lim_{x\to 0^+} (\cot x - \ln x)$

72. $\lim_{x\to 0^+} (\sin x)^x$

73. $\lim_{x\to\frac{\pi}{2}^-} (\tan x - \sec x)$

74. $\lim_{x\to 0} \dfrac{x - \sin x}{x - \tan x}$

75. $\lim_{x\to\infty} \left(\dfrac{x}{x-1}\right)^{x/2}$

76. $\lim_{x\to\frac{\pi}{2}^-} \left[\dfrac{\sin x}{\cos x} + \dfrac{1}{x - \frac{\pi}{2}}\right]$

77. $\lim_{x\to-\infty} x^2 e^x$

78. $\lim_{x\to 0} \dfrac{\sin x - x\cos x}{x - \sin x}$

79. $\lim_{x\to 3^-} \left[\dfrac{1}{\ln(x-2)} - \dfrac{1}{x-3}\right]$

80. $\lim_{x\to 0^+} (1+3x^2)^{1/x^2}$

81. $\lim_{x\to 1} \dfrac{x - e^{x-1}}{(x-1)^2}$

82. $\lim_{x\to\infty} x\left[\dfrac{\pi}{2} - \arctan x\right]$

83. $\lim_{x\to\infty} \left(\dfrac{x}{x+2}\right)^{x+1}$

84. $\lim_{x\to 0^+} x^{\tan x}$

85. $\lim_{x\to 0^+} (\ln x)\tan x$

86. $\lim_{x\to 0} (1+\sin x)^{1/(2x)}$

87. $\lim_{x\to\pi} \dfrac{\sin x + x - \pi}{1+\cos x}$

88. $\lim_{x\to 0^+} \left(\dfrac{2}{x} - \dfrac{1}{1-\cos x)}\right)$

89. $\lim_{x\to\frac{\pi}{2}^-} (2x\tan x - \pi\sec x)$

90. $\lim_{x\to\infty} (e^x + x)^{1/x}$

Answers:

1.	-2	2.	2	3.	0	4.	2	5.	$\frac{1}{2}$	6.	$\frac{1}{2}$	7.	-2	8.	1
9.	0	10.	$\frac{1}{3}$	11.	$\frac{1}{3}$	12.	$\frac{1}{e}$	13.	$\frac{1}{2}$	14.	$\frac{1}{e^2}$	15.	0	16.	0
17.	$\frac{1}{2}$	18.	2	19.	∞	20.	∞	21.	0	22.	$\frac{1}{e^2}$	23.	1	24.	$\frac{1}{e^2}$
25.	$\frac{1}{3}$	26.	a	27.	0	28.	e^2	29.	1	30.	0	31.	1	32.	2
33.	∞	34.	$e^{1/3}$	35.	$\frac{1}{e^4}$	36.	∞	37.	$-\frac{1}{2}$	38.	e	39.	-1	40.	0
41.	$\frac{1}{2}$	42.	$\frac{1}{e^3}$	43.	0	44.	$e^{1/3}$	45.	$\frac{1}{4}$	46.	0	47.	1	48.	e^2
49.	$\frac{1}{2}$	50.	∞	51.	$e^{-1/\pi}$	52.	1	53.	$-\frac{1}{\pi^2}$	54.	e^3	55.	0	56.	0
57.	0	58.	1	59.	$\frac{1}{2}$	60.	2	61.	e	62.	0	63.	e^2	64.	1
65.	e^2	66.	$\frac{3}{7}$	67.	e^6	68.	1	69.	0	70.	e	71.	∞	72.	1
73.	0	74.	$-\frac{1}{2}$	75.	$e^{1/2}$	76.	0	77.	0	78.	2	79.	$\frac{1}{2}$	80.	e^3
81.	$-\frac{1}{2}$	82.	1	83.	$\frac{1}{e^2}$	84.	1	85.	0	86.	$e^{1/2}$	87.	0	88.	$-\infty$
89.	-2	90.	e												