

# 1 Curve Sketching

For the each problem determine the following if applicable:

- (a)  $x$ - and  $y$ -intercepts
- (b) vertical and horizontal asymptotes
- (c) open intervals on which the function is increasing and decreasing
- (d) open intervals where the function is concave up and concave down
- (e) coordinates of all relative extrema
- (f) coordinates of all points of inflection
- (g) sketch the curve, showing required information

1.  $y = x^4 - 18x^2 + 56$

2.  $y = \frac{2x - x^2}{x^2 - 2x + 1}$

3.  $y = x^5 - 5x$

4.  $y = \frac{(x - 3)(x + 2)}{(x + 1)(x - 2)}$

5.  $y = x^4 - 4x^3$

6.  $y = x^3 - 9x^2 + 15x - 5$

7.  $y = x^3 - 3x^2 + 3$

8.  $y = 2x^3 - 12x^2 + 18x$

9.  $y = 2x^4 - x^2$

10.  $y = \frac{1}{4}(x^3 - \frac{3}{2}x^2 - 6x + 2)$

11.  $y = x^5 - 5x^4$

12.  $y = 8 - 9x + 6x^2 - x^3$

13.  $y = \frac{9x}{(3x + 1)^2}$   
 $y' = \frac{9(1 - 3x)}{(3x + 1)^3}$      $y'' = \frac{54(3x - 2)}{(3x + 1)^4}$

14.  $y = \frac{x^2}{x^2 - 1}$   
 $y' = \frac{-2x}{(x^2 - 1)^2}$      $y'' = \frac{2(3x^2 + 1)}{(x^2 - 1)^3}$

15.  $y = \frac{x}{(x + 1)^2}$   
 $y' = \frac{1 - x}{(x + 1)^3}$      $y'' = \frac{2(x - 2)}{(x + 1)^4}$

16.  $y = \frac{x^2 - 1}{x^3}$   
 $y' = \frac{-x^2 + 3}{x^4}$      $y'' = \frac{2x^2 - 12}{x^5}$

17.  $y = \frac{x + 1}{x^2}$   
 $y' = \frac{-x - 2}{x^3}$      $y'' = \frac{2x + 6}{x^4}$

18.  $f(x) = x^{2/3}(x - 5)$   
 $f'(x) = \frac{5(x - 2)}{3x^{1/3}}$      $f''(x) = \frac{10(x + 1)}{9x^{4/3}}$

19.  $y = \frac{1}{x^2 - x}$   
 $y' = \frac{1 - 2x}{(x^2 - x)^2}$      $y'' = \frac{2(3x^2 - 3x + 1)}{(x^2 - x)^3}$

20.  $y = x(16 - x^2)^{1/2}$   
 $y' = 2(16 - x^2)^{-1/2}(8 - x^2)$   
 $y'' = -2x(16 - x^2)^{-3/2}(24 - x^2)$

21.  $f(x) = 18(x^{-2} + x^{-1}) = \frac{18(x + 1)}{x^2}$

22.  $f(x) = x^{1/3}(x + 1)$   
 $f'(x) = \frac{4x + 1}{2x^{2/3}}$      $f''(x) = \frac{2(2x - 1)}{9x^{5/3}}$

23.  $f(x) = \frac{-6x}{x^2 + 1}$   
 $f'(x) = \frac{6(x + 1)(x - 1)}{(x^2 + 1)^2}$   
 $f''(x) = \frac{-12x(x^2 - 3)}{(x^2 + 1)^3}$

24.  $y = 5x^{2/3} - x^{5/3}$

25.  $f(x) = \frac{6 - 6x}{x^2}$   
 $f'(x) = \frac{6x - 12}{x^3}$      $f''(x) = \frac{36 - 12x}{x^4}$

26.  $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$   
 $f'(x) = -\frac{1}{x^2} - \frac{2}{x^3}$      $f''(x) = \frac{2}{x^3} + \frac{6}{x^4}$

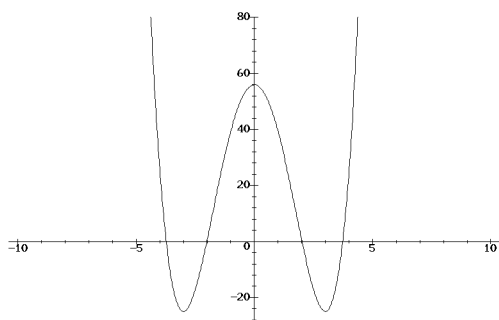
27.  $y = \frac{x^2 - 1}{x^3}$   
 $y' = \frac{3 - x^2}{x^4}$      $y'' = \frac{2(x^2 - 6)}{x^5}$

28.  $y = x^4 - 6x^2 + 8x + 10$   
 $\frac{dy}{dx} = 4(x - 1)^2(x + 2)$

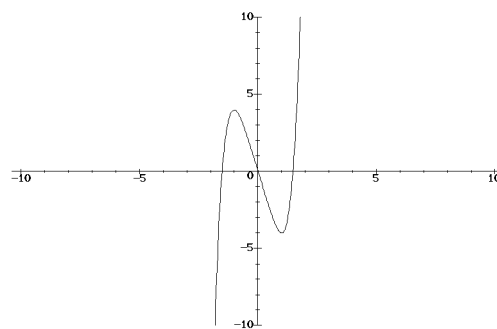
Answers: 1-4.

3.

1.



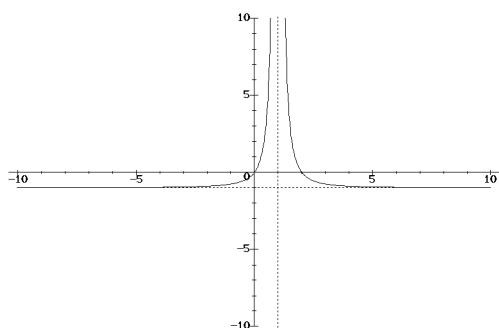
|                   |                                            |
|-------------------|--------------------------------------------|
| $x$ -intercepts   | $(\pm 2, 0); (\pm \sqrt{14}, 0)$           |
| $y$ -intercept    | $(0, 56)$                                  |
| H.A.              | none                                       |
| V.A.              | none                                       |
| Increasing on     | $(-3, 0), (3, \infty)$                     |
| Decreasing on     | $(-\infty, -3), (0, 3)$                    |
| Concave up on     | $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$ |
| Concave down on   | $(-\sqrt{3}, \sqrt{3})$                    |
| Rel. Max. at      | $(0, 56)$                                  |
| Rel. Min. at      | $(-3, -25); (3, -25)$                      |
| Inflection pt. at | $(-\sqrt{3}, 11); (\sqrt{3}, 11)$          |



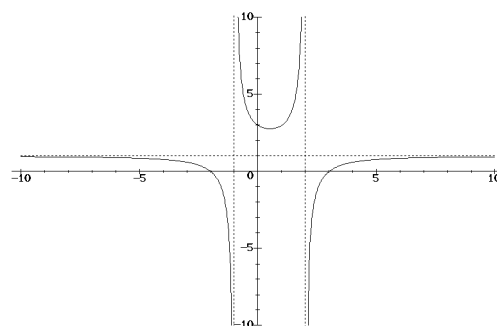
|                   |                                               |
|-------------------|-----------------------------------------------|
| $x$ -intercepts   | $(0, 0); (-\sqrt[4]{5}, 0); (\sqrt[4]{5}, 0)$ |
| $y$ -intercept    | $(0, 0)$                                      |
| H.A.              | none                                          |
| V.A.              | none                                          |
| Increasing on     | $(-\infty, -1), (1, \infty)$                  |
| Decreasing on     | $(-1, 1)$                                     |
| Concave up on     | $(0, \infty)$                                 |
| Concave down on   | $(-\infty, 0)$                                |
| Rel. Max. at      | $(-1, 4)$                                     |
| Rel. Min. at      | $(1, -4)$                                     |
| Inflection pt. at | $(0, 0)$                                      |

4.

2.



|                   |                             |
|-------------------|-----------------------------|
| $x$ -intercepts   | $(0, 0); (2, 0)$            |
| $y$ -intercept    | $(0, 0)$                    |
| H.A.              | $y = -1$                    |
| V.A.              | $x = 1$                     |
| Increasing on     | $(-\infty, 1)$              |
| Decreasing on     | $(1, \infty)$               |
| Concave up on     | $(-\infty, 1), (1, \infty)$ |
| Concave down on   | none                        |
| Rel. Max. at      | none                        |
| Rel. Min. at      | none                        |
| Inflection pt. at | none                        |



|                   |                                    |
|-------------------|------------------------------------|
| $x$ -intercepts   | $(-2, 0); (3, 0)$                  |
| $y$ -intercept    | $(0, 3)$                           |
| H.A.              | $y = 1$                            |
| V.A.              | $x = -1; x = 2$                    |
| Increasing on     | $(\frac{1}{2}, 2), (2, \infty)$    |
| Decreasing on     | $(-\infty, -1), (-1, \frac{1}{2})$ |
| Concave up on     | $(-1, 2)$                          |
| Concave down on   | $(-\infty, -1), (2, \infty)$       |
| Rel. Max. at      | none                               |
| Rel. Min. at      | $(\frac{1}{2}, \frac{25}{9})$      |
| Inflection pt. at | none                               |