Instructor: Dr. R.A.G. Seely

(Feb 2016)

Test 1

(version A)

Calculus III (Maths 201–DDB)

(Marks)

Note: Justify all your answers — don't make me guess your thoughts!

(2) 1. (a) Using the Binomial Theorem and the fact that $\ln(1-x) = -\int_0^x \frac{1}{1-t}\,dt$, derive the Maclaurin series for $f(x) = \ln(1-x)$. What is the interval of convergence of this series?

(2) (b) By choosing a suitable value for x, find a power series whose sum is $\ln 2$. Approximate the value of $\ln 2$ by using the first four terms of this series. *(Actually, there are two easily found values of x which will do; I will give more marks for choosing a value which gives a better approximation, or which is closer to the center of the interval of convergence.)*

(1) (c) Use your series for $f(x) = \ln(1-x)$ to find a general formula for the value of $f^{(n)}(0)$.

(4) 2. Construct a power series for $f(x) = \int_0^x \frac{1-\cos(t/2)}{t}\,dt$. What is its interval of convergence?

(4) Use this series to approximate $\int_0^{1/2} \frac{1-\cos(x/2)}{x}\,dx$ to 6 decimal places. Justify your error estimate.

(4) 3. Use the Binomial Series to derive a power series for $f(x) = 1/\sqrt{x}$ about $x = 4$.

(3) Use Taylor's Inequality to estimate the error in using the Taylor polynomial $T_4(x)$ to approximate $f(x)$ for $3 \le x \le 5$.

4. Consider the curve given by the following parametric equations: $\begin{cases} x = t^2 - 4t \\ y = t^3 - 3t^2 \end{cases}$

(6) (a) Find the x and y intercepts. Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, and all points with horizontal and vertical tangents. Find all points of inflection (where the curve changes concavity).
Sketch the graph, showing all these points. Indicate the direction of increasing t (the "orientation").

(5) (b) Find the area of the region in quadrant II below the curve (*i.e.* the region above the x axis, to the left of the y axis, and below the curve).

(4) (c) Set up (but don't evaluate) the integral needed to calculate the arc length of the part of the curve in quadrant II. (For a bonus mark, simplify the integral so it would be as easy to calculate as possible.)

(4) 5. Sketch $r = 3\sin\theta$ and $r = 1 + \sin\theta$ on the same axes. Find all points of intersection.

You do NOT need to evaluate the following two integrals.

(2) Set up the integral needed to calculate the area inside the first curve but outside the second (*i.e.* between the two curves).

(2) Set up the integral needed to calculate the perimeter of the second curve.

(3) 6. (a) Sketch the graph of $r^2 = 4\sin\theta$.

(2) (b) Find the area of one loop.

(2) (c) Set up the integral (but do not evaluate it!) needed to calculate the length of the perimeter (*i.e.* the arc length) of one loop.

(Total: 50)