**Calculus III** (Maths 201–DDB)

# Answers

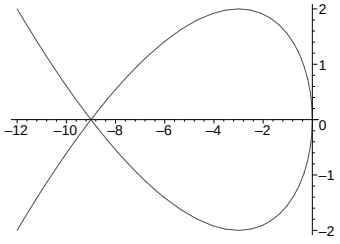
1. $\displaystyle\int_0^x \sin(t^2)\,dt = \frac{x^3}{3} - \frac{x^7}{7\cdot 3!} + \frac{x^{11}}{11\cdot 5!} - \frac{x^{15}}{15\cdot 7!} \pm \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!}$
   $\displaystyle\int_0^{1/2} \sin(x^2)\,dx \simeq \tfrac{1}{24} - \tfrac{1}{5376} \simeq 0.041480654$ (to within $\pm 10^{-6}$)
   since the $|\text{error}| \le \frac{1}{11\cdot 5!\cdot 2^{11}} < 3.70\times 10^{-7}$

2. $\displaystyle\frac{x}{\sqrt{1-x^2}} = x + \tfrac{1}{2}x^3 + \tfrac{1\cdot 3}{2^2\cdot 2!}x^5 + \tfrac{1\cdot 3\cdot 5}{2^3\cdot 3!}x^7 + \cdots = x + \sum_{n=1}^{\infty} \frac{(2n-1)!!}{2^n\cdot n!}x^{2n+1}$
   converges for $-1 < x < 1$.

3. $T_3(x) = 2 + \frac{1}{12}(x-8) - \frac{1}{288}(x-8)^2 + \frac{5}{20736}(x-8)^3$, so $\sqrt[3]{8.5} \simeq T_3(8.5) = 2.04082875$. For $|R_3(x)|$, we use the estimates: $M = \frac{80}{81}(7.5)^{-11/3} \simeq 6.1\times 10^{-4}$, and $|R_3(x)| \le \frac{M}{4!}(0.5)^4 \simeq 1.59\times 10^{-6}$. So $\sqrt[3]{8.5} = 2.04082875 \pm 1.59\times 10^{-6}$.

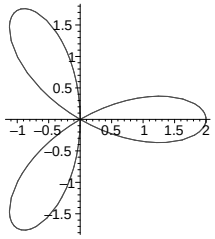
4. (a) $\frac{1}{2}$ (b) $\frac{1}{10}$

5. (a) $x$-int: $(0,0)$ at $t=0$; $(-9,0)$ at $t=\pm\sqrt{3}$
   $y$-int: $(0,0)$ again. $\frac{dy}{dx} = \frac{t^2-1}{2t}$ $\frac{d^2y}{dx^2} = \frac{t^2+1}{-12t^3}$
   VT: at $(0,0)$ $(t=0)$.
   HT: at $(-3,-2)$ $(t=-1)$ and $(-3,2)$ $(t=1)$.
   Graph at right: (orientation: start top left,
   go down right, around & up, finally back down to bottom left)

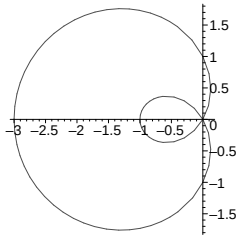



   (b) length: $12\sqrt{3}$ area: $\frac{72}{5}\sqrt{3}$

6. Graphs: (a)




   (b)




   The integrals: (a) $4\displaystyle\int_0^{\pi/6} \cos^2 3\theta\,d\theta$ (b) $\displaystyle\int_{\pi/3}^{5\pi/3} \sqrt{5-4\cos\theta}\,d\theta$