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Test 3
(version A)

Calculus III (Maths 201–DDB)

(Marks)

Justify all your answers — just having the correct answer is not sufficient.
Pace yourself — a rough guide is to spend not more than $2m$ minutes or so on a question worth m marks.

(8) 1. Suppose $F(x,y,z) = xz^2 - yz^3 + \cos(xy)$.

(a) Find the gradient of F at the point $P_0(0,8,-2)$.

(b) For the level surface (contour surface) $F(x,y,z) = 65$, find the equation of the tangent plane at P_0.

(c) On the level surface $F(x,y,z) = 65$ find $\dfrac{\partial z}{\partial y}$.

(d) If $z = f(x,y)$ is implicitly determined by the level surface $F(x,y,z) = 65$ and $f(0,8) = -2$, calculate $\nabla f(0,8)$, and use it (or the answer to (1b) above) to give an estimate of $f(0.1, 7.9)$.

(6) 2. Suppose $f(t)$ is differentiable, and let $z = yf(x^2 - y^2)$; show that

$$y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = \frac{xz}{y}$$

(6) 3. Let $f(x,y) = 3xy - x^3 + y^3$.

(a) Find and classify the critical points of $f(x,y)$.

(b) Find the absolute maximum and the absolute minimum values of $f(x,y)$ in the region bounded between the parabola $y = x^2$ and the line $y = 4$.

(6) 4. Use Lagrange Multipliers to find the maximum value of the product xyz for a point (x,y,z) which lies on the ellipsoid $x^2 + 2y^2 + z^2 = 3$.

(6) 5. Evaluate the following (change coordinates as appropriate):

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{1 - x^3}\, dx\, dy$$

(6) 6. Evaluate the double integral $\displaystyle\iint_{\mathcal{R}} e^{-(x^2+y^2)}\, dx\, dy$, where $\mathcal{R}$ is the entire xy plane.

(For a bonus mark, use this to derive the value of $\displaystyle\int_{-\infty}^{\infty} e^{-x^2}\, dx$.*)*

(6) 7. Sketch the solid region of integration for the following:

$$\int_0^{2\pi} \int_0^{\sqrt{3}} \int_{r/\sqrt{3}}^{\sqrt{4-r^2}} r\sqrt{r^2 + z^2}\, dz\, dr\, d\theta$$

Convert the integral to spherical coordinates. Evaluate the triple integral by whatever method you prefer.

(6) 8. Use the transformation $\{\, x = u + v\,,\ y = u - 3v \,\}$ to evaluate the integral $\displaystyle\iint_{\mathcal{R}} \sqrt{3x + y}\, dA$, where $\mathcal{R}$ is the region bounded by the lines $3x + y = 0$, $3x + y = 4$, $x - y = 0$, $x - y = 8$.

(Total: 50)