Instructor: Dr. R.A.G. Seely

Test 1
(version for practice)

Calculus III (Maths 201–DDB)

1. Consider the curve given by the following parametric equations.
$$\begin{cases} x = t^3 - 3t^2 \\ y = t^3 - 3t \end{cases}$$
Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, and all points with horizontal and vertical tangents. Find where the curve is concave up, and where it is concave down. Draw a rough sketch of the graph; be sure to include the part of the curve given by $-1 \le t \le 2$. (Hint: this part is a loop.)

   Set up, **but do not evaluate**, the integral needed to calculate the length of the loop given by this curve for $-1 \le t \le 2$.

2. Sketch $r = 1$ and $r = 1 + 2\cos\theta$ on the same axes. Find all points of intersection.

   Do NOT evaluate the following integrals.

   Set up the integrals needed to calculate the area inside the first curve but outside the second.

   Set up the integrals needed to calculate the perimeter of the second curve.

3. (a) Sketch the graph of $r = \sin 3\theta$.
   (b) Find the area of one loop.
   (c) Set up the integral (**but do not evaluate it!**) needed to calculate the length of the perimeter of the graph (*i.e.* the arc length of the entire curve).

4. Is the series
$$\sum_{n=1}^{\infty} (-1)^n \frac{\sin\left(\frac{1}{n}\right)}{\sqrt{n}}$$
absolutely convergent, is it conditionally convergent, or is it divergent? Justify your answer.

5. Find a power series for each of the following functions.
   (a) $x - \sin x$ (b) $x^3 \cos x$

   Use these power series to calculate $\lim_{x\to 0} \frac{x - \sin x}{x^3 \cos x}$. Verify your answer by calculating this limit one other way. (Hint: Long division is **not** necessary.)

6. Construct a power series for e^{-x^2} and so obtain a series for $\int_0^1 e^{-x^2}\, dx$. Using the first 4 non-zero terms of this series, approximate the value of the integral. Give a bound for the error of this approximation; justify your claim.

7. Use the Binomial Series to derive a power series for $f(x) = \frac{1}{x^2}$ about $x = 1$. Use Taylor's Inequality to estimate the error in using the third-degree Taylor polynomial $T_3(x)$ to approximate $f(x)$ for $0.8 \le x \le 1.2$.

Bonus question: Suppose $f(x) = \sum_{n=2}^{\infty} \frac{(-1)^n (x+5)^n}{n!\,\sqrt{n-1}}$; find $f^{(10)}(-5)$ ("without calculation").