Instructor: Dr. R.A.G. Seely Assignment 5

Calculus III (Maths 201–DDB)

1. Sketch the region of integration for the integral $\int_0^1 \int_y^1 \cos(x^2)\, dx\, dy$. Calculate the integral by reversing the order of integration.

2. Evaluate the following integrals:

   (a) $\displaystyle\int_0^2 \int_{y^2}^4 \frac{y^3}{\sqrt{y^4+x^3}}\, dx\, dy$ (b) $\displaystyle\int_0^{1/\sqrt{2}} \int_y^{\sqrt{1-y^2}} \frac{y^2}{\sqrt{y^2+x^2}}\, dx\, dy$

   (c) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \mathrm{e}^{-(x^2+y^2)}\, dy\, dx$ (d) $\displaystyle\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} z\, dz\, dx\, dy$

3. Let $R = \{(x,y) : 0 \le x \le 2,\ x^2 \le y \le 4\}$. Evaluate $\iint_R x\, \mathrm{e}^{y^2}\, dA$.

4. Let S be the solid region above the xy plane, inside the cylinder $x^2+y^2=4$, outside the cone $z^2 = x^2+y^2$. Evaluate (using spherical coordinates) $\iiint_S \frac{1}{x^2+y^2+z^2}\, dV$.

5. Find the volume of the solid lying in the first (positive) octant inside the cylinder $x^2+y^2=a^2$ and under the plane $z=y$.

6. Find the volume of the part of the sphere $x^2+y^2+z^2 = 4$ which lies inside the cylinder $x^2+y^2-2y=0$.

7. Find the volume of the region in the first octant bounded by the coordinate planes, the plane $y = 1-x$ and the surface $z = \cos\left(\frac{\pi x}{2}\right)$, $0 \le x \le 1$.

8. Find the volume of the region bounded by $z = \sqrt{x^2+y^2}$ and the paraboloid $z = 2-(x^2+y^2)$.

9. Find the volume of the region bounded by the surfaces $z = x^2+y^2$ and $z = 8-x^2-y^2$.

10. Set up the following integrals:

    (a) the double integral needed to find the area of the region common to both the cardioid $r = 4-4\sin\theta$ and the circle $r = 4\sin\theta$.

    (b) the triple integrals needed to find the volume of the solid inside the sphere $x^2+y^2+z^2=8$ and above the plane $z=2$, using each of the coordinate systems (Cartesian, cylindrical, spherical).

11. Let S be the solid region bounded by the surfaces $y = x^2+z^2$ and $y=4$. Write the triple integral $\iiint_S f(x,y,z)\, dV$ in the order given by $dV = dy\, dz\, dx$ and by $dV = dz\, dy\, dx$.

12.* Find the Jacobian of the transformation $u = x+y$, $v = x-y$. Use this transformation to evaluate $\iint_R \cos(x-y)\, dA$, where R is the region bounded by the lines $x-y=0$, $x-y=\frac{\pi}{2}$, $x+y=2$, $x+y=4$.