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Assignment 2

Calculus III (Maths 201–DDB)

Answer half the questions in each section. (Your choice, although I suggest you try the others, as they are good preparation for the tests and exam.)

1. For the following curves, given parametrically, find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$; locate all points where the tangent is horizontal or vertical (identify which), and sketch the graph. Describe the concavity. Where indicated, calculate the required area or length.

   (a) $\begin{cases} x = \frac{1}{3}t^3 \\ y = t^2 - 3t \end{cases}$ Area: between the curve and the x axis, for $0 \le t \le 4$.

   (b) $\begin{cases} x = 2\ln t \\ y = t + \frac{1}{t} \end{cases}$ Length: on the t-interval $[1, 5]$

   (c) $\begin{cases} x = \cos 2t \\ y = \cos t \end{cases}$ Area: the bounded region between the curve and the y axis.

   (d) $\begin{cases} x = \arcsin(t) \\ y = t^2 - 1 \end{cases}$ Area: between the curve and the x axis.
   (Hint: what is the maximum possible range of t?)

2. Find the Cartesian equations for the following curves, and sketch their graphs.

   (a) $\begin{cases} x = 2 - \cos t \\ y = 3\sin t + 1 \end{cases}$ (b) $\begin{cases} x = \sqrt{t} \\ y = 2t + 4 \end{cases}$ (c) $\begin{cases} x = \tan^{-1} t \\ y = t^2 - 1 \end{cases}$ (d) $\begin{cases} x = 2 - \frac{1}{t} \\ y = 2t + \frac{1}{t} \end{cases}$

3. (a) A bike tire with a radius of 30 cm has a stone stuck in the tread; if the tire travels along a path of 30 m, what is the length of the arched path of the stone?

   (b) Find the arc length of the curve with parametric equations $x = t^2\cos t,\ y = t^2\sin t,\ 0 \le t \le 2\pi$.

4. (a) Sketch the graphs of the following curves: $r = 2\cos\theta$ and $r = 2\cos 2\theta$. Find all points of intersection. Find the area of the region inside the first curve but outside the second.

   (b) Sketch the graphs of the following curves: $r = 2$ and $r^2 = 8\sin 2\theta$. Find all points of intersection. Find the area of the region common to both regions defined by the curves.

5. Do all parts of this question. For the limaçon $r = 1 - 2\sin\theta$:
   (a) sketch the curve;
   (b) find the tangent line to the curve at $\theta = \frac{\pi}{3}$;
   (c) find all points where the tangent is horizontal or vertical (identify which is which);
   (d) set up (but you need not evaluate) the integrals necessary to find the length of the inner loop, and the area between the inner and outer loops.

6. Find the Cartesian equations for the following curves, and sketch them.

   (a) $r = 2\cos\theta - \sin\theta$ (b) $r = \dfrac{1}{2\cos\theta - \sin\theta}$ (c) $r = 3\csc\theta$ (d) $r = 3 - 2\cos\theta$

7. Find the polar equations for the following curves, and sketch them.

   (a) $(x^2 + y^2)^2 - 4(x^2 - y^2) = 0$ (b) $xy = 9$

8. (a) Show that the tangent lines at the points of intersection of the circles $r = a\sin\theta$ and $r = a\cos\theta$ are perpendicular.

   (b) Given a curve expressed by parametric equations, show that $\dfrac{d^2y}{dx^2} = \dfrac{\frac{dx}{dt}\frac{d^2y}{dt^2} - \frac{dy}{dt}\frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^3}$

9. Sketch and name the curve $r = 2\sin^2\left(\frac{\theta}{2}\right)$.

   (a) Find its length, and

   (b) the area it encloses.