Calculus III (Maths 201–DDB)

1. For the following power series, find the radius and the interval of convergence.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{3n\sqrt{n}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n x^n}{(\log(n+1))^n}$ (c) $\displaystyle J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2}\left(\frac{x}{2}\right)^{2n}$

2. Given $\displaystyle f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n5^n}$ find the interval of convergence of the Taylor series expansions around $x = 5$ of the following:

   (a) $f(x)$, (b) $f'(x)$, (c) $\displaystyle\int_5^x f(t)\,dt$.

3. Find power series representations for each function and give the interval of convergence (don't bother checking the endpoints) for each.

   (a) $\sin^2 x$ (b) $\arctan(x^2)$ (c) $\dfrac{x^3}{e^x}$ (d) $\dfrac{e^x}{\cos x}$

   (e) $\left(\dfrac{x}{1+x}\right)^3$ (f) $\displaystyle\int_0^x \frac{\arctan t}{t}\,dt$ (g) $\dfrac{1}{\sqrt{1+x^2}}$

4. If $\displaystyle f(x) = \int_0^x \frac{1-e^{-t}}{t}\,dt$:

   (a) find a power series for $f(x)$ about $x = 0$;
   (b) find the interval of convergence of this series;
   (c) compute $f(0.4)$ to four decimal place accuracy justifying your answer.

5. (a) Find the Taylor series for $f(x) = \dfrac{3}{x^2 - x - 2}$ about $x = 1$. (Hint: partial fractions)
   (b) Find the interval of convergence of this series.
   (c) Use the series to compute $f^{(6)}(1)$

6. (a) Write the Maclaurin series for $f(x) = \sqrt{1+x}$ (use the Binomial Theorem). What is the interval of convergence?
   (b) Use the first 4 terms of the power series you just obtained to approximate $\sqrt{16.5}$.
   (N.B. Be careful about the interval of convergence!)

7. Find the value of n so that the error obtained by approximating $\sin x$ by the nth degree Maclaurin polynomial $T_n(x)$ on the interval $-.5 \le x \le .5$ will be less than 5×10^{-6}.

8. Find the Taylor polynomial $T_4(x)$ of degree 4 and the remainder $R_4(x)$ for $f(x) = \sqrt[4]{x}$ about $x = 16$. Use this to estimate $\sqrt[4]{15}$, and give the error bound for your estimation.