



## Calculus III (Maths 201–DDB)

1. Find the sum of the following series:

(a) 
$$\sum_{n=0}^{\infty} \left( \frac{1}{2^n} + \frac{1}{3^n} \right)$$

(b) 
$$\sum_{n=0}^{\infty} \left( \tan^{-1}(n+1) - \tan^{-1}(n+2) \right)$$

(c) 
$$\sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+2}}{3^n}$$

(d) 
$$\sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)}$$

2. Are the following series convergent? State the test used and show all the appropriate criteria are met.

(a) 
$$\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$$

(b) 
$$\sum_{n=1}^{\infty} \frac{n^2 + 1}{n(2n-1)}$$

(c) 
$$\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$$

(d) 
$$\sum_{n=1}^{\infty} \frac{n^n}{n!}$$

(e) 
$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

(f) 
$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{\ln(1+n)}}$$

(g) 
$$\sum_{n=1}^{\infty} \frac{\sin(1/n)}{(\ln(1+n))^2}$$

(h) 
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{n+1}}$$

3. Are the following absolutely convergent, conditionally convergent, or divergent? (Again, give a full justification of your response.)

(a) 
$$\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$$

(b) 
$$\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{(n+1)^n}$$

(c) 
$$\sum_{n=1}^{\infty} \frac{\sin n}{n^2 + 1} \sqrt{n}$$

(d) 
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(1 + 1/n)^{n^2}}$$

Optional “warm-up” questions: Are these C (if appropriate, specify AC or CC) or D?

1. 
$$\sum_{n=1}^{\infty} \frac{\cos n}{2^n}$$

2. 
$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$$

3. 
$$\sum_{n=1}^{\infty} \frac{1}{n + \sqrt{n}}$$

4. 
$$\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2 + 1}$$

5. 
$$\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$$

6. 
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n(n+1)}}$$

7. 
$$\sum_{n=1}^{\infty} \frac{2^n}{3^n + 5}$$

8. 
$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}$$

9. 
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{e^n}$$

10. 
$$\sum_{n=1}^{\infty} \frac{n^2}{e^n}$$

11. 
$$\sum_{n=1}^{\infty} \frac{2^n(n+1)}{n!}$$

12. 
$$\sum_{n=1}^{\infty} \frac{1}{n(1 + \ln n)}$$