

Cal III Assgt 00 Solutions

① (a) $= \int \frac{1+\cos 2t}{2} dt = \frac{1}{2}t + \frac{1}{4}\sin 2t + C = \frac{1}{2}t + \frac{1}{2}\sin t \cos t + C$

(b) $= \int (1-\cos^2 x) \sin x dx = \int (u^2-1) du = \frac{1}{3}\cos^3 x - \cos x + C$

$\begin{cases} u = \cos x \\ du = -\sin x \end{cases}$

(c) $= \int_0^{2\pi} \sqrt{\frac{1-\cos t}{2}} \cdot \frac{1+\cos t}{1+\cos t} dt = \int_0^{2\pi} \frac{|\sin t|}{\sqrt{1+\cos t}} dt = 2 \int_0^{\pi} \frac{\sin t}{\sqrt{1+\cos t}} dt =$ [This is actually improper-but ok.] $\begin{cases} u = 1+\cos t \\ du = -\sin t dt \end{cases}$
 $= -4\sqrt{1+\cos t} \Big|_0^{\pi} = 4\sqrt{2}$ [Alt: use $1-\cos t = 2\sin^2 \frac{t}{2}$]

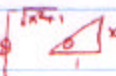
(d) $= \frac{1}{2}(x^2+1)\arctan x - \frac{1}{2}x + C$

$\begin{cases} u = \arctan x & du = \frac{dx}{1+x^2} \\ v = x \end{cases}$

(e) $= \int \frac{dx-2}{\sqrt{4-(x-2)^2}} = \int d\theta = \arcsin\left(\frac{x-2}{2}\right) + C$

(f) $= \int \sec^3 \theta d\theta = \sec \theta \tan \theta - \int \sec \theta \tan^2 \theta d\theta$
 $= \sec \theta \tan \theta - \int \sec^3 \theta d\theta + \int \sec \theta d\theta$

$\begin{cases} u = \sec \theta & du = \sec \theta \tan \theta d\theta \\ v = \tan \theta \end{cases}$



$\begin{cases} x^2-4x = (x-2)^2-4 \\ \frac{2}{\sqrt{4-x^2}} \cdot x-2 & \frac{x-2}{\sqrt{4-x^2}} = 2\sin \theta \\ \frac{2}{\sqrt{4-x^2}} \cdot x-2 & \frac{d(x-2)}{\sqrt{4-x^2}} = 2\cos \theta d\theta \end{cases}$

so $\int \sec^3 \theta d\theta = \frac{1}{2}\sec \theta \tan \theta + \frac{1}{2}\ln|\sec \theta + \tan \theta|$

so (f) $= \frac{1}{2}x\sqrt{x^2+1} + \frac{1}{2}\ln|x+\sqrt{x^2+1}| + C$

(g) $= x^2\sqrt{x^2+1} - \frac{2}{3}(x^2+1)^{3/2} + C = \frac{1}{3}(x^2+1)^{3/2} - (x^2+1)^{1/2} + C$

$\begin{cases} u = \frac{1}{2}x^2 & du = x dx \\ v = \sqrt{x^2+1} & dv = \frac{2x dx}{\sqrt{x^2+1}} \\ w = x-2 & dw = dx \end{cases}$

(h) $= \int \frac{dx}{(x+2)(x-3)} = -\frac{1}{5}\ln|x+2| + \frac{1}{5}\ln|x-3| + C$

$\frac{1}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$ (etc)

(i) Fact: $1+(x-\frac{1}{4x})^2 = (x+\frac{1}{4x})^2$, so (i) $= \int_2^4 (x+\frac{1}{4x}) dx$
 $= \left[\frac{x^2}{2} + \frac{1}{4}\ln x \right]_2^4 = 6 + \frac{1}{4}\ln 2$

② Area $= \int_0^6 ((2x)-(x^2-4x)) dx = \int_0^6 (6x-x^2) dx = \left[3x^2 - \frac{x^3}{3} \right]_0^6 = 36$

③ (i) $V = \int_0^9 \pi x dx = \frac{\pi}{2} \cdot 81 = \frac{81\pi}{2}$

(ii) $V = \int_0^9 2\pi x^{3/2} dx = \frac{4\pi}{5} 9^{5/2} = \frac{972}{5}\pi$

④ $= \lim_{x \rightarrow 0} \frac{1-\frac{1}{1+x^2}}{5x^2} = \lim_{x \rightarrow 0} \frac{1}{3(1+x^2)} = \frac{1}{3}$