Cal III Assgt 00 Solutions

① (a) $= \int \frac{1+\cos 2t}{2}\,dt = \frac{1}{2}t + \frac{1}{4}\sin 2t + C = \frac{1}{2}t + \frac{1}{2}\sin t\cos t + C$

(b) $= \int (1-\cos^2 x)\sin x\,dx = \int (u^2-1)\,du = \frac{1}{3}\cos^3 x - \cos x + C$ $\left[u = \cos x,\ du = -\sin \right.$

(c) $= \int_0^{2\pi} \sqrt{\frac{1-\cos t}{1}\cdot\frac{1+\cos t}{1+\cos t}}\,dt = \int_0^{2\pi} \frac{|\sin t|}{\sqrt{1+\cos t}}\,dt = 2\int_0^{\pi}\frac{\sin t}{\sqrt{1+\cos t}}\,dt =$ [This is actually improper - but ok.] $\left[u = 1+\cos t,\ du = -\sin t\,d \right.$

$= -4\sqrt{1+\cos t}\,\Big]_0^{\pi} = 4\sqrt{2}$ [**Alt:** use $1-\cos t = 2\sin^2\frac{t}{2}$]

(d) $= \frac{1}{2}(x^2+1)\arctan x - \frac{1}{2}x + C$ $\left[u = \arctan x,\ du = \frac{dx}{1+x^2},\ dv = ,\ v = \right.$

(e) $= \int \frac{d(x-2)}{\sqrt{4-(x-2)^2}} = \int d\theta = \arcsin\left(\frac{x-2}{2}\right) + C$ $\left[x^2-4x = (x-2)^2 - 4;\ x-2 = 2\sin\theta,\ d(x-2) = 2\cos\theta\,d\theta,\ \sqrt{4x-x^2} = 2\cos\theta \right.$

(f) $= \int \sec^3\theta\,d\theta = \sec\theta\tan\theta - \int \sec\theta\tan^2\theta\,d\theta$ $\left[u = \sec\theta,\ dv = \sec^2\theta\,d\theta,\ du = \sec\theta\tan\theta\,d\theta,\ v = \tan\theta \right.$

$= \sec\theta\tan\theta - \int\sec^3\theta\,d\theta + \int\sec\theta\,d\theta$

so $\int\sec^3\theta\,d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta|$

so (f) $= \frac{1}{2}x\sqrt{x^2+1} + \frac{1}{2}\ln\left|x+\sqrt{x^2+1}\right| + C$

(g) $= x^2\sqrt{x^2+1} - \frac{2}{3}(x^2+1)^{3/2} + C = \frac{1}{3}(x^2+1)^{3/2} - (x^2+1)^{1/2} + C$ $\left[u = \frac{1}{2}x^2,\ du = x\,dx,\ dv = \frac{2x\,dx}{\sqrt{x^2+1}},\ v = 2\sqrt{x^2+1},\ dv = 2x\sqrt{x^2+1}\,dx,\ v = \frac{2}{3}(x^2+1)^{3/2} \right.$

(h) $= \int\frac{dx}{(x+2)(x-3)} = -\frac{1}{5}\ln|x+2| + \frac{1}{5}\ln|x-3| + C$ $\left[\frac{1}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}\ (\text{etc}) \right]$

(i) Fact: $1 + \left(x - \frac{1}{4x}\right)^2 = \left(x + \frac{1}{4x}\right)^2$, so (i) $= \int_2^4 \left(x + \frac{1}{4x}\right)dx$

$= \frac{x^2}{2} + \frac{1}{4}\ln x\,\Big]_2^4 = 6 + \frac{1}{4}\ln 2$

② area $= \int_0^6 \left((2x) - (x^2-4x)\right)dx = \int_0^6 (6x - x^2)\,dx = 3x^2 - \frac{x^3}{3}\Big]_0^6 = 36$

③ (i) $V = \int_0^9 \pi x\,dx = \frac{\pi}{2}\cdot 81 = \frac{81\pi}{2}$

(ii) $V = \int_0^9 2\pi x^{3/2}\,dx = \frac{4\pi}{5}9^{5/2} = \frac{972}{5}\pi$

④ $= \lim_{x\to 0}\frac{1 - \frac{1}{1+x^2}}{3x^2} = \lim_{x\to 0}\frac{1}{3(1+x^2)} = \frac{1}{3}$