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Test 3
(version "for practice")

Cal II (S) (Maths 201–NYB)

Integrate:

1. $\displaystyle\int \frac{\cos^3 x\,dx}{(\sin(x))^{2/3}}$
2. $\displaystyle\int x \operatorname{arcsec} x\,dx$
3. $\displaystyle\int_0^1 \sin^2\left(\tfrac{\pi x}{2}\right)\cos^2\left(\tfrac{\pi x}{2}\right)\,dx$
4. $\displaystyle\int x^2 \arctan x\,dx$
5. $\displaystyle\int \frac{x^2+1}{\sqrt{x+2}}\,dx$
6. $\displaystyle\int_{\pi/3}^{\pi/2} \sin^3 x\cos 2x\,dx$
7. $\displaystyle\int_2^{2\sqrt{2}} \frac{\sqrt{x^2-4}}{x}\,dx$
8. $\displaystyle\int \arcsin\sqrt{x}\,dx$
9. $\displaystyle\int \frac{(1+x\sec x)^2}{x}\,dx$
10. $\displaystyle\int_{\ln\sqrt{2}}^{\ln 2} \sqrt{2\,\mathrm{e}^{2x}-4}\,dx$
11. $\displaystyle\int \frac{\mathrm{e}^{-\sqrt{x}}\cos\sqrt{x}}{\sqrt{x}}\,dx$
12. $\displaystyle\int_0^{\pi/2} (1+\sin x)\cos^3 x\,dx$
13. $\displaystyle\int \frac{\ln(x^2+1)}{x^3}\,dx$
14. $\displaystyle\int \frac{4x^2}{\sqrt{2x+3}}\,dx$
15. $\displaystyle\int \frac{6x^2+23x-53}{(x-1)(x+2)(x-3)}\,dx$
16. $\displaystyle\int \frac{dx}{(9x^2+5)^{3/2}}$
17. $\displaystyle\int \sec^4\left(\tfrac{x}{2}\right)\tan^2\left(\tfrac{x}{2}\right)\,dx$
18. $\displaystyle\int \frac{-x^3+5x^2+5x-3}{x^3(x+3)}\,dx$

Limits:

1. $\displaystyle\lim_{x\to 2^+}\left(\frac{8}{x^2-4}-\frac{x}{x-2}\right)$
2. $\displaystyle\lim_{x\to 0}\frac{x\,\mathrm{e}^{3x}-x}{1-\cos 2x}$
3. $\displaystyle\lim_{x\to 0^+}\left(\mathrm{e}^x+x\right)^{1/2x}$
4. $\displaystyle\lim_{x\to 0^+}\left(\cos 3x\right)^{2/x}$

Improper integrals:

1. $\displaystyle\int_{-\infty}^3 \frac{dx}{x^2+9}$
2. $\displaystyle\int_{-\infty}^{\infty} \mathrm{e}^{3x}\,dx$
3. $\displaystyle\int_4^6 \frac{dx}{(5-x)^{2/5}}$
4. $\displaystyle\int_{-3}^3 \frac{dx}{x\sqrt[3]{x}}$
5. $\displaystyle\int_1^{\infty} \frac{dx}{x(x+1)}$
6. $\displaystyle\int_0^{\infty} \frac{x\,dx}{\mathrm{e}^x}$
7. $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{dx}{\sin x}$
8. $\displaystyle\int_{-1}^1 \frac{\mathrm{e}^x\,dx}{1-\mathrm{e}^x}$

Sequences: For each sequence, determine whether or not it is convergent.
[OPTIONAL: For each sequence, write the first 4 terms. Determine if the sequence is monotonic (↗ or ↘?)]

1. $\displaystyle\frac{5^n}{n^2\,n!}$
2. $\displaystyle\frac{2n}{2n-1}$
3. $\displaystyle\frac{n!}{n^3\,2^n}$
4. $\displaystyle\frac{n}{3n+1}$

Series — what's the sum?:

1. $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k^2+k}$
2. $\displaystyle\sum_{k=3}^{\infty}\frac{2}{k^2-2k}$
3. $\displaystyle\sum_{n=0}^{\infty}\frac{1+2^n}{3^n}$
4. $\displaystyle\sum_{n=0}^{\infty}\frac{5^n-2^n}{7^n}$

Infinite series (converge/diverge?):

1. $\displaystyle\sum_{n=1}^{\infty}\frac{n^2+1}{\sqrt{n^5+2n^2}}$
2. $\displaystyle\sum_{n=0}^{\infty}\mathrm{e}^{-n}$
3. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\sqrt{n-1}}$
4. $\displaystyle\sum_{k=1}^{\infty}\frac{k}{e^{k^2}}$
5. $\displaystyle\sum_{n=1}^{\infty}\frac{n^2\,5^n}{(2n)!}$
6. $\displaystyle\sum_{n=1}^{\infty}\frac{n^3-5}{n^5+7n^2-5}$
7. $\displaystyle\sum_{n=1}^{\infty}\left(2-\frac{n}{n^2+4}\right)$
8. $\displaystyle\sum_{n=1}^{\infty}\frac{\sec^2(n)}{\sqrt[3]{n}}$
9. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n^3+1}{3n^3-2n}\right)^n$
10. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\sqrt{\ln n}}$
11. $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k^2+k}$
12. $\displaystyle\sum_{n=0}^{\infty}\frac{1+2^n}{3^n}$
13. $\displaystyle\sum_{k=2}^{\infty}\frac{1}{k^2-k}$
14. $\displaystyle\sum_{n=0}^{\infty}\frac{2+3^n}{5^n}$
15. $\displaystyle\sum_{k=1}^{\infty}\left(1+\frac{1}{2k}\right)^k$
16. $\displaystyle\sum_{k=1}^{\infty}\frac{k^3-2}{k^5-2k^2+6}$
17. $\displaystyle\sum_{k=1}^{\infty}\frac{\csc^2(k)}{\sqrt[3]{k}}$
18. $\displaystyle\sum_{k=1}^{\infty}\left(\frac{k+1}{2k-3}\right)^k$

19. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^{3/2}}$
20. $\displaystyle\sum_{k=1}^{\infty} \left(1 - \frac{1}{2k}\right)^k$
21. $\displaystyle\sum_{k=1}^{\infty} \frac{k2^k}{(2k)!}$
22. $\displaystyle\sum_{k=1}^{\infty} \frac{\sec^2(k)}{\sqrt{k}}$
23. $\displaystyle\sum_{k=1}^{\infty} \frac{k^2-2}{k^5-3k^2+7}$
24. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$
25. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^4+2n}{3n^4-5n^2+1}\right)^n$
26. $\displaystyle\sum_{k=1}^{\infty} \frac{k^4 4^k}{(k+1)!}$
27. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3n^4-n}{n^4-2n^2+5}\right)^n$
28. $\displaystyle\sum_{k=1}^{\infty} \frac{k^3 3^k}{(k+3)!}$
29. $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n^3+5}}{n^2+2n-1}$
30. $\displaystyle\sum_{k=1}^{\infty} \frac{\cos^2 k}{\sqrt[3]{k^4}}$
31. $\displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{2}{3n^2}\right)^n$
32. $\displaystyle\sum_{k=1}^{\infty} \frac{k!\,4^k}{(2k+1)!}$
33. $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n^3+5n^2}}{n^3+2n-1}$
34. $\displaystyle\sum_{k=1}^{\infty} \frac{2}{\cos^2 k\,\sqrt[3]{k}}$
35. $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n^2+5}}{n^3+2n-1}$
36. $\displaystyle\sum_{k=1}^{\infty} \frac{\csc^2 k}{\sqrt[3]{k}}$
37. $\displaystyle\sum_{n=1}^{\infty} \left(2 - \frac{1}{2n}\right)$
38. $\displaystyle\sum_{k=1}^{\infty} \frac{\sin^2(k)}{k^3}$
39. $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k+1}{3k-2}\right)^k$
40. $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{1}{2n}\right)$
41. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3-5}{\sqrt{n^7+5n^2-5}}$
42. $\displaystyle\sum_{k=1}^{\infty} \frac{(2k)!}{k^5 2^k}$
43. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt[3]{\ln n}}$
44. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^4+n}{2n^4-3n^2}\right)^n$
45. $\displaystyle\sum_{n=1}^{\infty} \frac{n^4+n}{2n^4-3n^2}$

Series (AC, CC, D?):

1. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n^5}{(n+3)!}$
2. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\, n}{\sqrt[5]{2n^9+6n+1}}$
3. $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{k!}{(2k)!}$
4. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k k}{(2k+1)^2}$
5. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k k}{(3k-10)^2}$
6. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k k^3}{k!}$
7. $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{k\sqrt{\ln k}}$
8. $\displaystyle\sum_{n=0}^{\infty} \cos(n\pi)\, \mathrm{e}^{-n}$
9. $\displaystyle\sum_{n=0}^{\infty} \frac{\sin(n)}{\sqrt{n^3}}$
10. $\displaystyle\sum_{k=2}^{\infty} (-1)^k \frac{\sqrt{k}}{\ln k}$
11. $\displaystyle\sum_{n=1}^{\infty} \frac{\sec n\pi}{n}$
12. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\, n^2}{\sqrt[3]{2n^4+n+1}}$

Interval of convergence:

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n\,(x+2)^n}{3^n}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{n\,x^n}{3^n}$
3. $\displaystyle\sum_{k=1}^{\infty} \frac{k(x+3)^k}{2^k}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n\,x^n}{n}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{n\,5^n}$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{n\,(x-5)^n}{2^n}$
7. $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n}{n^5\,5^n}$
8. $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n}{5^n\sqrt[3]{n}}$

Taylor:

1. Find the Maclaurin series for $f(x) = \ln(x+1)$. Write the first 4 non-zero terms explicitly, and express the n^{th} term in terms of a general formula. Write the series in sigma notation. What is the radius of convergence?
2. Use the series in (1) for $\ln(x+1)$ to derive a series for $\dfrac{1}{x+1}$ (centered at $x = 0$). Does this series converge at $x = 1$ to the value 1/2? (Justify your answer.)
3. For the function $f(x) = \mathrm{e}^{-3x}$: (a) find the first four terms of the Maclaurin series for $f(x)$; (b) find the n^{th} term, and express the series in Σ notation. (c) What is the radius of convergence for this series?
4. Do the same for $f(x) = \mathrm{e}^{x/3}$; for $f(x) = \cos 2x$; for $f(x) = \sin x/2$.
5. Find the Taylor series for $f(x) = \sqrt{x}$ about $x = 1$. (Follow the same pattern as the questions above.)