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Test 2
(version "for practice")

Cal II (S) (Maths 201–NYB)

(Warm-up!) Integrate:

1. $\int \sin^4 x\,dx$
2. $\int \frac{\csc^4 x\,dx}{\sqrt[3]{\cot x}}$
3. $\int \arctan x\,dx$
4. $\int e^{2x}\cos x\,dx$
5. $\int \frac{(\ln x)^2\,dx}{x}$
6. $\int (\ln x)^2\,dx$
7. $\int \frac{x^3-1}{x^3-2x^2}\,dx$
8. $\int \frac{x^5}{\sqrt{x^2+9}}\,dx$
9. $\int \frac{dx}{2x^2-5x-3}$
10. $\int \frac{dx}{\sqrt{1-x^2}\arcsin(x)}$
11. $\int \text{arcsec} x\,dx$
12. $\int \frac{x\,dx}{(x^2+9)^2}$
13. $\int \frac{dx}{(x^2+9)^2}$
14. $\int \frac{x^2+x-1}{(x+1)(x^2-1)}\,dx$
15. $\int \frac{\tan^5 x\,dx}{\sec^2 x}$
16. $\int_1^2 \ln x\,dx$
17. $\int \sin^2 x\cos^2 x\,dx$
18. $\int \sec^4(\frac{x}{2})\tan^2(\frac{x}{2})\,dx$
19. $\int \sin^2(\frac{x}{2})\cos^2(\frac{x}{2})\,dx$
20. $\int x^2e^{3x}\,dx$
21. $\int_{\frac{1}{\sqrt{2}}}^1 \arcsin x\,dx$
22. $\int \frac{e^x\,dx}{1+e^{2x}}$
23. $\int \sec^4 x\tan^3 x\,dx$
24. $\int \frac{\cos^3 x\,dx}{\sin^4 x}$
25. $\int_{\sqrt[4]{2}}^{\sqrt{2}} \frac{dx}{x\sqrt{x^4-1}}$
26. $\int e^{2x}\sin x\,dx$
27. $\int \frac{dx}{x^4-1}$
28. $\int \frac{dx}{(7-6x-x^2)^{\frac{5}{2}}}$
29. $\int_0^1 \frac{3x-1}{\sqrt{4-x^2}}\,dx$
30. $\int \frac{x^2-1}{x^2+2x}\,dx$
31. $\int \frac{x^5}{\sqrt{x^2+9}}\,dx$
32. $\int \frac{\cos^5 x\,dx}{\sin^4 x}$
33. $\int_{\sqrt[6]{2}}^{\sqrt[3]{2}} \frac{dx}{x\sqrt{x^6-1}}$
34. $\int \cos(\ln x)\,dx$
35. $\int_{\frac{1}{\sqrt{2}}}^1 \arccos x\,dx$
36. $\int \sin^2(\frac{x}{4})\cos^2(\frac{x}{4})\,dx$
37. $\int \frac{\cos^3 x\,dx}{\sin^6 x}$
38. $\int x^2e^{2x}\,dx$
39. $\int \sin(\ln x)\,dx$
40. $\int e^x\sin 2x\,dx$
41. $\int \frac{2x^4-1}{x^2(x-2)}\,dx$
42. $\int \frac{6x^2-16x+3}{(x-2)^2(2x+1)}\,dx$
43. $\int \sqrt[4]{x-3}\,(x^2+6)\,dx$
44. $\int_1^{\sqrt{3}} \frac{\sqrt{x^2+1}}{x^4}\,dx$
45. $\int \sin^2(\frac{x}{3})\cos^2(\frac{x}{3})\,dx$
46. $\int x^3e^{2x}\,dx$
47. $\int \frac{e^{2x}\,dx}{1+e^{4x}}$
48. $\int_0^1 x\arctan(x)\,dx$
49. $\int \sec^3 x\tan^4 x\,dx$
50. $\int \frac{\ln x}{\sqrt{x}}\,dx$
51. $\int \frac{\sqrt{4x^2-9}}{x}\,dx$
52. $\int \frac{4x-3}{x^2+16}\,dx$
53. $\int \frac{\sin^2\theta}{\cos\theta}\,d\theta$
54. $\int x\,\text{arcsec}\,x\,dx$
55. $\int \frac{x^3}{\sqrt{25-x^2}}\,dx$
56. $\int \frac{4x^3-6x^2-1}{2x^2-5x-3}\,dx$
57. $\int \frac{x^2-2x}{x^3-3x^2}\,dx$

Limits:

1. $\lim_{x\to\infty} \dfrac{\arctan x}{5-e^{-x}}$
2. $\lim_{x\to 0} \dfrac{x}{\ln(x+1)}$
3. $\lim_{n\to\infty} \left(1-\dfrac{3}{n}\right)^{\frac{n}{4}}$
4. $\lim_{\theta\to 0} \dfrac{\theta-\tan\theta}{\sin^2\theta}$
5. $\lim_{x\to 0} \dfrac{\arctan x}{\tan 2x}$
6. $\lim_{x\to 0^+} x\ln(x^2)$
7. $\lim_{x\to 1^+} \left(\dfrac{1}{\ln x}+\dfrac{1}{1-x}\right)$
8. $\lim_{n\to\infty} n^{1/n}$
9. $\lim_{x\to\infty} x\ln\left(1-\dfrac{1}{x}\right)$
10. $\lim_{x\to 1} \dfrac{x-1}{\ln x}$
11. $\lim_{x\to\infty} \dfrac{4-e^{-x}}{\arctan x}$
12. $\lim_{n\to\infty} \left(1-\dfrac{4}{n}\right)^{\frac{n}{3}}$
13. $\lim_{\theta\to 0} \dfrac{\theta-\sin\theta}{\tan^2\theta}$
14. $\lim_{x\to 0} \dfrac{\arcsin x}{\sin 2x}$
15. $\lim_{x\to 0^+} x\ln x$
16. $\lim_{x\to 1^+} \left(\dfrac{1}{\ln x}-\dfrac{1}{x-1}\right)$
17. $\lim_{n\to\infty} n^{1/n^2}$
18. $\lim_{n\to\infty} \left(1+\dfrac{2}{n}\right)^{-3n}$
19. $\lim_{x\to 0} \dfrac{\ln(x+1)-x}{x^2}$
20. $\lim_{x\to 0} \dfrac{\sec x}{x}$
21. $\lim_{x\to\infty} x(e^{2/x}-1)$
22. $\lim_{x\to 0} \dfrac{\arctan x}{x}$
23. $\lim_{x\to 0} (1+\sin 3x)^{(1/x)}$
24. $\lim_{x\to 0} \dfrac{x-\ln(x+1)}{x^2}$
25. $\lim_{x\to 0} \dfrac{\arcsin x}{x}$
26. $\lim_{x\to\infty} x(e^{1/x}-1)$
27. $\lim_{x\to 0} (1+\sin 5x)^{(1/x)}$
28. $\lim_{x\to 0^+} \dfrac{\cos x}{x}$
29. $\lim_{x\to 0^+} \dfrac{1-\cos x}{x}$
30. $\lim_{x\to 0^+} (\cos x)^{(1/x^2)}$

Improper integrals:

1. $\int_0^{\frac{\pi}{2}} \dfrac{dx}{\sin x}$
2. $\int_0^3 \dfrac{dx}{\sqrt[3]{x-1}}$
3. $\int_0^\infty xe^{-x}\,dx$
4. $\int_2^\infty \dfrac{dx}{x^2+x}$
5. $\int_0^4 \dfrac{x\,dx}{x-2}$
6. $\int_2^\infty \dfrac{dx}{x^2-1}$
7. $\int_0^{\frac{\pi}{2}} \dfrac{dx}{\cos x}$
8. $\int_0^3 \dfrac{dx}{\sqrt[3]{x-2}}$
9. $\int_0^{\frac{\pi}{2}} \dfrac{dx}{\tan x}$
10. $\int_{-\infty}^0 xe^{-x}\,dx$
11. $\int_0^3 \dfrac{dx}{(x-2)^{4/3}}$
12. $\int_1^\infty \dfrac{\mathrm{e}^{-\sqrt{x}}}{\sqrt{x}}\,dx$

Volumes, areas, arclengths:

1. Find the area between:
   (a) $y=x+\dfrac{8}{x}$ and $y=\dfrac{x}{2}+5$; (b) the curves $x-y=0$ and $y^2-x=2$ and above the x axis;
   (c) x^3-2x and $3x$; (d) $x-3y=0$ and $x+y=y^3$ above the x axis;
   (e) $y=4-\dfrac{4}{x^2}$ and $y=x^2-1$, for $x\geq 0$.
2. Find the volume of the solid obtained when the region between the curves $y=2x-x^2$ and $y=x^3$ above the x-axis is rotated (a) about the y-axis; (b) about the x-axis.
3. Set up definite integral which expresses the area of the region (or regions) bounded by the graphs of $y=x^3-4x+7$ and $y=7$.
4. Find the volume of the solid generated when the region between the given curves is rotated about (a) the x-axis and (b) the y-axis.
   (a) $y=x^2$ and $y=2x$, (b) $y=x^4$ and $y=8x$,
   (c) $y=x^3-2x+1$ and $y=2x+1$, (d) $y=\sqrt{1-3x^2}$ and $y=x$.
5. Find the arclength of the following curves:
   (a) $y=1+4x\sqrt{x}$ from $x=0$ to $x=1$ (b) $y=\frac{1}{2}x^2-\frac{1}{4}\ln(x)$ from $x=2$ to $x=4$
   (c) $y=\ln(\cos(x))$ from $x=0$ to $x=\frac{\pi}{3}$ (d) $y^3=x^2$ from $(-1,1)$ to $(1,1)$
6. Use an appropriate arclength integral to verify that the circumference of a circle with radius r is $2\pi r$.

Differential equations:

1. Find the particular solution of the following differential equation (satisfying the given initial conditions): give your answers as explicit functions $y = f(x)$.
   (a) $xyy' = \ln x$ where $y(1) = 2$ (b) $\ln(y)\, y' = xy$ where $y(2) = 1$

2. Find the general solution of the following differential equations:
   (a) $y' = 1 + y^2 - 2x - 2xy^2$ (b) $y' = 2y(5 - y)$
   (c) $y' = y^2 \sin x$ (d) $y' = \sqrt{xy}$
   (e) $\mathrm{e}^x \sin^2(y)\, y' = x \sec(y)$

3. A plant spills a toxic solution into the ground at a rate of 5 tons per year. These solvents do not all stay in the ground: each year $\frac{1}{10}$ of the total amount of solvents evaporates into the air. Find a formula for the total amount $A(t)$ of solvents in the ground after t years, assuming that initially there are none. Eventually (as $t \to \infty$), how many tons of solvents will accumulate in the ground?
   This question is Q7 on the May 2015 exam.

4. A certain (fictional!) species of worms is found to have a growth rate where the rate of increase in length is proportional to the difference between its current length and its eventual adult length (in cm). If the eventual length is 6 cm, the initial measured length is 2 cm, and 1 day later its length is 4 cm, then find the differential equation that expresses this relationship, and so find the functional equation $\ell = f(t)$ expressing the length ℓ in terms of time t.
   For this question, I'll give you the final answer: $\ell(t) = 6 - 4 \cdot 2^{-t}$. Check this for yourself!