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Quiz 4
(version P)

Calculus II (Maths 201–NYB)

1. Evaluate:
   (a) $\int x \tan^2 x \; dx$ (b) $\int x \sin^3 x \; dx$ (c) $\int \sin 8x \cos 5x \; dx$
   (d) $\int \tan x \sec^3 x \; dx$ (e) $\int \ln\left(\sqrt[3]{x}\,\right) \; dx$ (f) $\int \frac{1}{(2+x)^2}\sqrt{\frac{1-x}{2+x}} \; dx$
   (g) $\int \sqrt{1+\sqrt{x}} \; dx$ (h) $\int \frac{x^2}{(3+4x-4x^2)^{3/2}} \; dx$ (i) $\int \frac{2x-1}{(3+4x-4x^2)^{3/2}} \; dx$
   (j) $\int \frac{x^2+x+1}{(x-1)^2(x^2+1)} \; dx$ (k) $\int \sqrt{1-4x^2} \; dx$

2. Evaluate the limits
   (a) $\lim_{x\to 0} \cot 2x \sin 6x$ (b) $\lim_{x\to\infty} \Big(\ln(x^7-1) - \ln(x^5-1)\Big)$ (c) $\lim_{x\to 0} (\mathrm{e}^x - x)^{1/x^2}$

3. Find the values of c for which the area of the region bounded by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is 576.

4. Solve the differential equation $(4+x^2)^2\, y' = -2\pi x(1+y^2)$ with $y(0) = \frac{1}{\sqrt{3}}$. Express y as a function of x.

5. For a series $\sum_{n=0}^{\infty} a_n$, you are told that $s_n = \dfrac{n-1}{n+1}$. Find a formula for a_n and the sum $\sum_{n=0}^{\infty} a_n$.

6. Does the following series converge or diverge? $\sum_{n=0}^{\infty} (-1)^n \dfrac{n+4^n}{n+6^n}$

7. Determine whether the following series converges absolutely, or conditionally, or diverges?
   $\sum_{n=1}^{\infty} (-1)^n \ln\left(\dfrac{n+2}{n+1}\right)$ Hint: 

8. Find the 4^{th} degree Maclaurin polynomial for $y = \sqrt{1+x}$.
   For a bonus: what is the Maclaurin series? (*I.e.* give the general form in Σ notation.)