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Quiz 2
(version for practice)

Calculus II (Maths 201–NYB)

Warm-up review:

(a) $\displaystyle\int \frac{\cos^5 x\,dx}{\sin^4 x}$ (b) $\displaystyle\int \frac{x^2+x-1}{(x+1)(x^2-1)}\,dx$ (c) $\displaystyle\int \frac{\csc^4 x\,dx}{\sqrt[3]{\cot x}}$

(d) $\displaystyle\int \sin^2(\tfrac{x}{4})\cos^2(\tfrac{x}{4})\,dx$ (e) $\displaystyle\int \frac{x^2-2x}{x^3-3x^2}\,dx$ (f) $\displaystyle\int \frac{dx}{(x^2+9)^2}$

(g) $\displaystyle\int \frac{dx}{x^4-1}$ (h) $\displaystyle\int \sec^4 x\tan^3 x\,dx$ (i) $\displaystyle\int \frac{dx}{\sqrt{1-x^2}\arcsin(x)}$

(j) $\displaystyle\int \frac{4x^3-6x^2-1}{2x^2-5x-3}\,dx$ (k) $\displaystyle\int x^2e^{2x}\,dx$ (l) $\displaystyle\int \frac{x^3+x^2+x-1}{x^4+x^2}\,dx$

(m) $\displaystyle\int_{\frac{1}{\sqrt{2}}}^{1} \arcsin x\,dx$ (n) $\displaystyle\int \cos(\ln x)\,dx$ (o) $\displaystyle\int_1^{\sqrt{3}} \frac{\sqrt{x^2+1}}{x^4}\,dx$

Now the actual "practice quiz":

1. Evaluate:
   (a) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{\sin x}$ (b) $\displaystyle\int_0^3 \frac{dx}{\sqrt[3]{x-1}}$ (c) $\displaystyle\int_1^\infty \frac{\mathrm{e}^{-\sqrt{x}}}{\sqrt{x}}\,dx$
   (d) $\displaystyle\int_0^4 \frac{x\,dx}{x-2}$ (e) $\displaystyle\int_2^\infty \frac{dx}{x^2-1}$
2. Evaluate:
   (a) $\displaystyle\lim_{x\to0} \frac{\arctan x}{\tan 2x}$ (b) $\displaystyle\lim_{x\to0^+} x\ln(x^2)$ (c) $\displaystyle\lim_{x\to1^+}\left(\frac{1}{\ln x}+\frac{1}{1-x}\right)$
   (d) $\displaystyle\lim_{x\to0^+} x^{1/(\ln(\mathrm{e}^x-1))}$ (e) $\displaystyle\lim_{x\to0}(1+\sin 3x)^{(1/x)}$ (f) $\displaystyle\lim_{x\to0^+}(\cos x)^{(1/x^2)}$
3. Find the area between:
   (a) $y=x^3-2x$ and $y=3x$; (b) $x-3y=0$ and $x+y=y^3$ above the x axis;
4. Find the volume of the solid obtained when the region between the curves $y=2x-x^2$ and $y=x^3$ above the x-axis is rotated (a) about the y-axis; (b) about the x-axis.
5. Find the volume of the solid obtained when the region between $y=\sin x$, $y=0$, $x=0$, and $x=\pi$ is rotated about the line $x=5$.
6. Find the arclength of the following curves on the given intervals:
   (a) $y=\frac{1}{6}x^5+\frac{1}{10}x^{-3}$ on $[1,2]$ (b) $y=\frac{1}{3}x^{3/2}-\sqrt{x}$ on $[1,4]$
7. Solve the following differential equations; write your answer as explicit functions $y=f(x)$ (with a suitable constant c).
   (a) $\displaystyle\frac{dy}{dx}=\frac{y}{x^2-1}$ (b) $y'=y(1-y)$ (c) $\displaystyle\frac{dy}{dx}=\frac{x\sec y}{1+x^2}$
8. Solve the following initial value problems.
   (a) $y'=xy^2$; $y(0)=1$ (b) $y'=\mathrm{e}^{-y}\sqrt{x}$; $y(1)=0$
9. Suppose that the rate of spread of a virus is proportional to the product of the fraction of the population infected and the fraction uninfected by the virus. When data collection started, only 20% were infected, but 2 days later 50% were infected. How many days (from when data collection started) will it take for 80% to be infected by this virus? *(Let $t=0$ days when data collection started.)*