Instructor: Dr. R.A.G. Seely | Quiz 2

(version for practice)

**Calculus II** (Maths 201–NYB)

Warm-up review:

(a) $-\frac{1}{3}\csc^3 x + 2\csc x + \sin x + C$

(b) $-\frac{1}{2}\frac{1}{x+1} + \frac{3}{4}\ln|x+1| + \frac{1}{4}\ln|x-1| + C$

(c) $-\frac{3}{2}(\cot x)^{2/3} - \frac{3}{8}(\cot x)^{8/3} + C$

(d) $\frac{x}{8} - \frac{1}{8}\sin x + C$

(e) $\frac{2}{3}\ln|x| + \frac{1}{3}\ln|x-3| + C$

(f) $\frac{1}{18}\frac{x}{x^2+9} + \frac{1}{54}\arctan(\frac{x}{3}) + C$

(g) $\frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| - \frac{1}{2}\arctan(x) + C$

(h) $\frac{1}{4}\tan^4 x + \frac{1}{6}\tan^6 x + C \quad = \quad \frac{1}{6}\sec^6 x - \frac{1}{4}\sec^4 x + C$

(i) $\ln|\arcsin(x)| + C$

(j) $x^2 + 2x + \frac{53}{7}\ln|x-3| + \frac{3}{7}\ln|2x+1| + C$

(k) $\frac{1}{4}\,\mathrm{e}^{2x}(2x^2 - 2x + 1) + C$

(l) $\frac{1}{x} + \ln|x| + 2\arctan(x) + C$

(m) $\frac{\pi}{2} - \frac{4+\pi}{4\sqrt{2}}$

(n) $\frac{x}{2}(\sin(\ln x) + \cos(\ln x)) + C$

(o) $\frac{2\sqrt{2}}{3} - \frac{8}{9\sqrt{3}}$

1. Improper integrals:
   (a) diverges (b) $\frac{3}{2}(2^{2/3} - 1)$ (c) $2/\mathrm{e}$
   (d) diverges (e) $\frac{1}{2}\ln 3$
2. Limits:
   (a) 1/2 (b) 0 (c) 1/2
   (d) e (e) $\mathrm{e}^3$ (f) $\mathrm{e}^{-1/2}$
3. Areas:
   (a) 25/2 (b) 4
4. Volumes: (a) $13\pi/30$ (b) $41\pi/105$
5. Volume: $2\pi(10-\pi)$
6. Arclengths:
   (a) 1261/240 (b) 10/3
7. Differential equations:
   (a) $y = c\sqrt{1-x}/\sqrt{1+x}$ (b) $y = c\,\mathrm{e}^x/(1 + c\,\mathrm{e}^x)$ (c) $y = \arcsin\left(c + \frac{1}{2}\ln(x^2+1)\right)$
8. Initial value problems:
   (a) $y = 2/(2-x^2)$ (b) $y = \ln(\frac{2}{3}x^{3/2} + \frac{1}{3})$
9. Viral problem: 4 days
   **Key steps in the solution:** First, the equation: $\frac{dp}{dt} = kp(1-p)$, $p(0) = 0.2 = \frac{1}{5}$, $p(2) = 0.5 = \frac{1}{2}$.
   The solution (to start with): $p = \dfrac{1}{1 + 4\,\mathrm{e}^{-kt}}$
   Find $k$ using $p(2) = \frac{1}{2}$: $k = \ln 2$.
   So (since $\mathrm{e}^{\ln(2^{-t})} = 2^{-t}$), $p = \dfrac{1}{1 + 4\cdot 2^{-t}}$.
   Finally, solve $p(t) = .8$ to get $t = 4$.