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Quiz 2
(version A)

Calculus II (Maths 201–NYB)

(2×2) 1. Evaluate:

(a) $\displaystyle\int_0^2 \frac{x\,dx}{x-1}$ (b) $\displaystyle\int_1^\infty x\,\mathrm{e}^{-x}\,dx$

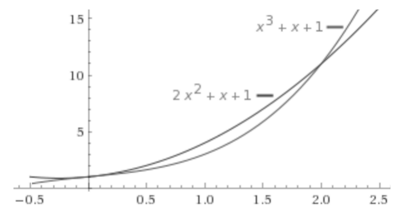
(3×2) 2. Set up the integrals necessary to find:

(a) the area between the curves $y = x^3 + x + 1$ and $y = 2x^2 + x + 1$;

(b) the volume when this region is rotated about the line $x = 0$;

(c) the volume when this region is rotated about the line $y = 15$;

(You do not have to evaluate these integrals!)



(3) 3. Find the arclength of the following curve on the given interval:

$y = \frac{1}{2}x^2 - \ln(\sqrt[4]{x})$ on $[1, 2]$

(2) 4. Solve the following initial value problem:

$\frac{dy}{dx} = y(x+1),\quad y(0) = 1$

Alternate: Find the area between the following curves: $y = x + 1$, $y = \frac{1}{x}$, $x = 1$, and $x = 2$.

Answers

1. Improper integrals:

(a) diverges (b) $2/\mathrm{e}$

2. Area and volume integrals:

(a) $\displaystyle\int_0^2 (2x^2 - x^3)\,dx := 4/3$

(b) $\displaystyle 2\pi\int_0^2 x(2x^2 - x^3)\,dx := 16\pi/5$

(c) $\displaystyle \pi\int_0^2 [(14 - x^3 - x^2)^2 - (14 - 2x^2 - x)^2]\,dx := \frac{2816\pi}{105}$

3. length $\displaystyle s = \int_1^2 \left(x + \frac{1}{4x}\right) dx = \frac{3}{2} + \frac{1}{4}\ln 2$

4. $\int \frac{dy}{y} = \int (x+1)\,dx$ so $\ln|y| = \frac{1}{2}x^2 + x + C$, but at $(0, 1)$: $0 = 0 + C$, so $y = \mathrm{e}^{\frac{1}{2}x^2+x}$

Alternate: area $= \int_1^2 (x + 1 - \frac{1}{x})\,dx = \frac{5}{2} - \ln 2$