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Test 1
(version for practice)

Cal I (S) (Maths 201–NYA)

1. For each of the following functions, find the derivative $f'(x)$ *using the limit definition.*

   (a) $f(x) = 5x + 7$ (b) $f(x) = \sqrt{x+1}$ (c) $f(x) = \dfrac{1}{x^2}$ (d) $f(x) = 3x^2 + 5$

   (e) $f(x) = \dfrac{3}{x-2}$ (f) $f(x) = 25x - 10$ (g) $f(x) = x^3 + 8$ (h) $f(x) = \dfrac{1}{\sqrt{x}}$

2. For each of the following functions, find the derivative $\frac{dy}{dx} = f'(x)$ *using the derivative formulas.*

   (a) $f(x) = \sqrt[5]{x^{42}}$ (b) $f(x) = 7x - 3$ (c) $f(x) = 7\sqrt[5]{x} - \dfrac{2}{x^5}$ (d) $f(x) = x^5 - \dfrac{2}{5x^3} + \sqrt[3]{x^4}$

   (e) $f(x) = 5x^{12} - 3x^5 + \frac{1}{4}x + 4.5$ (f) $f(x) = \left(\dfrac{1}{x^{42}} + 42\right)\left(3x^4 - 2x + 9\right)$

   (g) $f(x) = (6x^{\frac{2}{5}} - 5x^2 + \pi)(2\sqrt{x} + x^2)$ (h) $f(x) = \dfrac{2x^5 - 7x^3 + 21}{15}$

   (i) $f(x) = (3x^2 + 9x^{\frac{3}{7}} - 2\pi)(3\sqrt[4]{x} - x^3)$ (j) $f(x) = \left(\dfrac{1}{x^{24}} + 42\right)\left(4x^3 - 9x + 2\right)$

   (k) $f(x) = \left(\dfrac{1}{x^{13}} - 26\right)\left(3x^4 - 8x + 5\right)$ (l) $f(x) = \dfrac{4x^5 - 2x^3 + 21}{7}$

   (m) $f(x) = \dfrac{5}{x^6} + 7\sqrt[3]{x}$ (n) $f(x) = \dfrac{(12x^3 - 5)\csc x}{\left(\frac{5}{x} - 13x^2 + e\right)}$

   (o) $f(x) = (4x^2 + 5x^{\frac{4}{9}} - \pi^3)(3\sqrt[4]{x} - x^4)$ (p) $y = \dfrac{5x^9 - \frac{1}{x} + 1}{9x^2 - 3x + 5}$

   (q) $y = \dfrac{(2x^3 - 4)^9}{(5x + 3x^2 + 1)^7}$ (r) $y = \left(3x^6 - 4x^2 + 21\right)^{13}(4x - 11)^5$

   (s) $y = \dfrac{(5x^9 - \frac{1}{x} + 1)^7}{\sqrt[3]{9x^2 - 3x + 5}}$ (t) $y = \cos(x^5 + 2)\tan(x^3 - 8)$

   (u) $y = \dfrac{4x^5}{7} - \dfrac{3}{x^2} + 11\sqrt[3]{x} - 2x^6 - \dfrac{2}{5}$ (v) $y = \dfrac{(2x^4 - 5)\sec x}{\sqrt{5x + 3x^2 + 1}}$

   (w) $y = \cot^4\left(\sqrt[3]{x^5 - 7x^4 + 17}\right)$ (x) $y = \left(8x^3 - 5x + 1\right)^8\left(3\sqrt{x} - x^2 - 7\right)^3$

3. MORE: (Use logarithmic differentiation where appropriate.)

   (a) $y = \sec^4\left(\sqrt[5]{x^3 - 4x^7 + 71}\right)$ (b) $y = \csc(x^3 - 8)\tan(x^5 + 2)$

   (c) $y = \mathrm{e}^{x^3 - x}\csc\left(3x^6 - 4x^2 + 21\right)$ (d) $y = \mathrm{e}^{5x^3 - 4x^2}\ln\left(3x^6 - \dfrac{4}{x^2} + 21\right)$

   (e) $y = \sin(x^5 + 2)\cot(x^3 - 8)$ (f) $y = \sec^5\left(\sqrt[3]{x^5 - 7x^4 + 17\ln x}\right)$

   (g) $y = \mathrm{e}^{2x^3+1} - 3\ln(x^2 + 1)$ (h) $y = \ln^4\left(\cot\left(x^5 - 7x^4 + 17\right)\right)$

   (i) $y = (\ln x)^{x^2+1}$ (j) $y = e^{x^3+1}\ln(3x^5 - \sec x)$

   (k) $y = \log_7\left(x^2 + \ln x\right)$ (l) $y = \dfrac{\ln(4x^3 - 1)}{\sqrt[7]{3x - 5}}$

   (m) $y = 7^{2x} - 3\log_5(x^4 + 1)$ (n) $y = \tan^6\left(\ln(3x^8 - \mathrm{e}^{x^3-8})\right)$

   (o) $y = \ln\left(\dfrac{\sqrt[3]{3x^4 + 2x - 5}}{2^x\cos^3(x^4 - 10)}\right)$ (p) $y = \ln\left(\dfrac{2^x\cos^3(x^4 - 10)}{\sqrt[3]{3x^4 + 2x - 5}}\right)$

4. For each of the following equations, find the first and second derivatives $\frac{dy}{dx}, \frac{d^2y}{dx^2}$:
   (a) $x^2y^5 - 3y^2 = \ln(xy^2) - 3$ (b) $y^3 - x^5 = \ln(x^4y^5) + 23$ (c) $\sin^3 y - x^5 = \ln(3x^4y^5) + \pi^2$

5. Find the slope and the equation of the tangent line to each of the following curves at the given point.
   (a) $y = 5x^3 - 3x^2$ at $x = 1$ (b) $y = 3x^6 - 5x^2$ at $x = 1$ (c) $y = 5x^2 - 2x^3$ at $x = 2$

6. Find the equations of the lines tangent to the curve $y = x^3 - 3x^2 - 15x + 7$ which are parallel to the straight line $9x - y + 3 = 0$.

7. Find all values of x at which the graph of the following function has a horizontal tangent line: $y = 3x^4 - 10x^3 - 9x^2 + 5$.

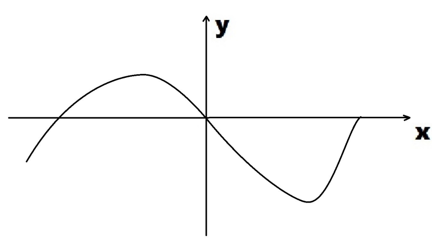
8. Find the values of x for which the lines tangent to the curve $y = x^3 - 3x^2 - 15x + 7$ are normal (*i.e.* at right angles) to the straight line $9x - y + 3 = 0$.

9. In analyzing the energy consumption of a robotic device, the following equation is used: $v = \frac{z}{\alpha(1 - z^2) - \beta}$ where α and β are fixed constant values. Find $\frac{dv}{dz}$.

10. The volume V of a sphere is related to the radius r by the formula $V = \frac{4}{3}\pi r^3$. At what rate is the volume changing relative to the radius when the radius of the sphere is 15 mm?

11. A ball is thrown up in the air with an initial velocity of 8 m/sec. The position function is $x(t) = -4.9t^2 + v_0 t + x_0$, where v_0 is the initial velocity, and x_0 the initial position. What are the velocity and acceleration functions? At what time t does the ball hit the ground? At what time t does it "turn around" (*i.e.* stop going up and start coming back down)? (*Hint:* what is the ball's velocity at that moment?) (Give your answer correct to 2 decimal places.)

12. Here is a graph of the derivative $f'(x)$ of a function $f(x)$. Draw a possible sketch of the function $y = f(x)$ itself.



Now, suppose the graph above is a graph of the function $y = f(x)$: then draw a possible sketch of the derivative $f'(x)$.

[*Note: another way to ask this question is to start with the given graph of the derivative $f'(x)$, and ask for graphs for both the function $f(x)$ and its second derivative $f''(x)$.*]