Cal I (S) (Maths 201–NYA)

1. Calculate the following limits.

   (a) $\lim\limits_{x\to 1} \dfrac{x^2+x-2}{x^2+2x-3}$ (b) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$ (c) $\lim\limits_{x\to 3} \dfrac{x^2+2x-15}{x-3}$

   (d) $\lim\limits_{x\to 0} \dfrac{\frac{1}{x+4}-\frac{1}{4}}{x}$ (e) $\lim\limits_{x\to 7} \dfrac{2x-1}{x-7}$ (f) $\lim\limits_{x\to 7} \dfrac{x-7}{2x-1}$

2. For each of the following functions, find the derivative $f'(x)$ *using a limit definition.*

   (a) $f(x)=5x+7$ (b) $f(x)=\sqrt{x+1}$ (c) $f(x)=\dfrac{1}{x^2}$ (d) $f(x)=x^2-3x+1$

   (e) $f(x)=3x^2+5$ (f) $f(x)=\dfrac{3}{x-2}$ (g) $f(x)=\dfrac{2}{x-5}$ (h) $f(x)=\sqrt{2-x}$

   (i) $f(x)=25x-10$ (j) $f(x)=x^3+8$ (k) $f(x)=32x+6$ (l) $f(x)=x^3-27$

   (m) $f(x)=23x+5$ (n) $f(x)=\dfrac{1}{\sqrt{x}}$ (o) $f(x)=\dfrac{1}{x+1}$ (p) $f(x)=x^2-3x+27$

3. For each of the following functions, find the derivative $f'(x)$ *using the derivative formulas.*

   (a) $f(x)=5x+7$ (b) $f(x)=\sqrt[5]{x}$ (c) $f(x)=\dfrac{1}{x^{42}}$ (d) $f(x)=x^2-3x+1$

   (e) $f(x)=\sqrt[5]{x^{42}}$ (f) $f(x)=7x-3$ (g) $f(x)=7\sqrt[5]{x}-\dfrac{2}{x^5}$ (h) $f(x)=x^5-\dfrac{2}{5x^3}+\sqrt[3]{x^4}$

   (i) $f(x)=5x^{12}-3x^5+\frac{1}{4}x+4.5$ (j) $f(x)=\left(\dfrac{1}{x^{42}}+42\right)(3x^4-2x+9)$

   (k) $f(x)=(6x^{\frac{2}{5}}-5x^2+\pi)(2\sqrt{x}+x^2)$ (l) $f(x)=\dfrac{2x^5-7x^3+21}{15}$

   (m) $f(x)=3-7x$ (n) $f(x)=\dfrac{3}{x^7}+5\sqrt[4]{x}$ (o) $f(x)=\frac{1}{5}x-4x^{17}+2x^3+4.5$

   (p) $f(x)=(3x^2+9x^{\frac{3}{7}}-2\pi)(3\sqrt[4]{x}-x^3)$ (q) $f(x)=\left(\dfrac{1}{x^{24}}+42\right)(4x^3-9x+2)$

   (r) $f(x)=\dfrac{3x^4-6x^5+12}{5}$ (s) $f(x)=3+7x$ (t) $f(x)=3x^{17}-4x^5+\frac{1}{9}x+7.5$

   (u) $f(x)=\left(\dfrac{1}{x^{13}}-26\right)(3x^4-8x+5)$ (v) $f(x)=\dfrac{4x^5-2x^3+21}{7}$

   (w) $f(x)=\dfrac{5}{x^6}+7\sqrt[3]{x}$ (x) $f(x)=\dfrac{(12x^3-5)\csc x}{\left(\frac{5}{x}-13x^2+e\right)}$ (y) $f(x)=(4x^2+5x^{\frac{4}{9}}-\pi^3)(3\sqrt[4]{x}-x^4)$

4. For each of the following functions, find the derivative $\frac{dy}{dx}$.

   (a) $y=4x^5-3x^2+\sqrt[3]{x}-\dfrac{2}{5}$ (b) $y=7x^3+22x-\dfrac{1}{x^8}+\sqrt{45}$ (c) $y=(8x^3-5x+1)(3\sqrt{x}-x^2-7)$

   (d) $y=\dfrac{5x^9-\frac{1}{x}+1}{9x^2-3x+5}$ (e) $y=\dfrac{(2x^3-4)^9}{(5x+3x^2+1)^7}$ (f) $y=(3x^6-4x^2+21)^{13}(4x-11)^5$

   (g) $y=\dfrac{4x^7-\frac{1}{x}+1}{7x^3+5x-3}$ (h) $y=4x^5-5x^3+\sqrt[4]{x}-\dfrac{3}{7}$ (i) $y=5x^4+13x-\dfrac{1}{x^7}+\sqrt{34}$

   (j) $y=\dfrac{(5x^3-4)^8}{(7x+3x^4+1)^5}$ (k) $y=\dfrac{5x^7-\frac{1}{x^3}+11}{29x^4-3x^2+5}$ (l) $y=(6x^5+3x-1)(5\sqrt{x}-x^3+6)$

   (m) $y=\dfrac{(2x^3-4)\sin x}{(5x+3x^2+1)}$ (n) $y=\sec(2x^7-7x^2)$ (o) $y=(4x^6-4x^9+65)^{24}(7x+13)^4$

   (p) $y=7x^3-\dfrac{2x}{9}-\dfrac{6}{x^7}+\sqrt{e}$ (q) $y=e^x\cos x\,(3x^6-4x^2+21)$ (r) $y=(8x^3-5x+1)(3\sqrt{x}-x^2-7)$

   (s) $y=\dfrac{(5x^9-\frac{1}{x}+1)^7}{\sqrt[3]{9x^2-3x+5}}$ (t) $y=\cos(x^5+2)\tan(x^3-8)$ (u) $y=\dfrac{4x^5}{7}-\dfrac{3}{x^2}+11\sqrt[3]{x}-2x^6-\dfrac{2}{5}$

   (v) $y=\dfrac{(2x^4-5)\sec x}{\sqrt{5x+3x^2+1}}$ (w) $y=\cot^4\left(\sqrt[3]{x^5-7x^4+17}\right)$ (x) $y=(8x^3-5x+1)^8(3\sqrt{x}-x^2-7)^3$

5. For each of the following functions, find the derivative $\frac{dy}{dx}$.
   (a) $y = \cot(2x^7 - 7x^2)$ (b) $y = \dfrac{9x^2 - 3x + 5}{5x^9 - \frac{1}{x} + 1}$ (c) $y = \dfrac{5x^4}{3} - \dfrac{7}{x^6} + 23\sqrt[5]{x} - 7x^3 - \dfrac{5}{2}$
   (d) $y = \sec^4\left(\sqrt[5]{x^3 - 4x^7 + 71}\right)$ (e) $y = \csc(x^3 - 8)\tan(x^5 + 2)$ (f) $y = e^{x^3 - x} \csc\left(3x^6 - 4x^2 + 21\right)$
   (g) $y = \left(8\sqrt{x} - x^3 - 7\right)\left(3x^2 - 7x + 1\right)$ (h) $y = \dfrac{4\pi^5}{7} - \dfrac{\sqrt{2}}{x^6} + \dfrac{11}{\sqrt[3]{x}} - 2x^6 - \dfrac{2x^{11}}{5}$
   (i) $y = \dfrac{\left(5x + 3x^2 + 1\right)\sin x}{(2x^3 - 4)}$ (j) $y = \csc(6x^5 - 7x^2 + 1)$ (k) $y = e^{x^5 - 2x}\cos\left(6x^3 - 2x^4 + 12\right)$
   (l) $y = \left(\dfrac{8}{x^3} - 5x^2 + 1\right)\left(3\sqrt[4]{x} - x^2 - \dfrac{1}{7}\right)$ (m) $y = e^{5x^3 - 4x^2}\ln\left(3x^6 - \dfrac{4}{x^2} + 21\right)$
   (n) $y = \dfrac{5x^9 - \frac{1}{\sqrt{x}} + 1}{9x^2 - \frac{3}{x} + 5}$ (o) $y = \sin(x^5 + 2)\cot(x^3 - 8)$ (p) $y = \sec^5\left(\sqrt[3]{x^5 - 7x^4 + 17\ln x}\right)$

6. Find the slope and the equation of the tangent line to each of the following curves at the given point.
   (a) $y = 5x^3 - 3x^2$ at $x = 1$ (b) $y = 3x^6 - 5x^2$ at $x = 1$ (c) $y = 5x^2 - 2x^3$ at $x = 2$
   (d) $y = \sqrt{x^2 + 5} - 2x$ at $(2, -1)$ (e) $y = \sqrt{x^2 - 5} - x$ at $(3, -1)$ (f) $y = \sqrt[3]{x^2 - 1} + 2x$ at $(3, 8)$

7. Find the equations of the lines tangent to the curve $y = x^3 - 3x^2 - 15x + 7$ which are parallel to the straight line $9x - y + 3 = 0$.

8. Find all values of x at which the graph of the following function has a horizontal tangent line: $y = 3x^4 - 10x^3 - 9x^2 + 5$.

9. Find the values of x for which the lines tangent to the curve $y = x^3 - 3x^2 - 15x + 7$ are parallel to the straight line $9x - y + 3 = 0$.

10. Find the values of x for which the lines tangent to the curve $y = x^3 - 3x^2 - 18x + 7$ are parallel to the straight line $9x + y + 3 = 0$.

11. Find the values of x for which the lines tangent to the curve $y = x^3 - 3x^2 - 15x + 7$ are normal (*i.e.* at right angles) to the straight line $9x - y + 3 = 0$.

12. In analyzing the energy consumption of a robotic device, the following equation is used: $v = \dfrac{z}{\alpha(1 - z^2) - \beta}$ where α and β are fixed constant values. Find $\dfrac{dv}{dz}$.

13. The volume V of a sphere is related to the radius r by the formula $V = \frac{4}{3}\pi r^3$. At what rate is the volume changing relative to the radius when the radius of the sphere is 15 mm?

14. The surface area A of a sphere is related to the radius r by the formula $A = 4\pi r^2$. At what rate is the surface area changing relative to the radius when the radius of the sphere is 6 mm?

15. A ball is thrown up in the air with an initial velocity of 8 m/sec. The position function is $x(t) = -4.9t^2 + v_0 t + x_0$, where v_0 is the initial velocity, and x_0 the initial position. What are the velocity and acceleration functions? At what time t does the ball hit the ground? At what time t does it "turn around"? (*I.e.* stop going up and start coming back down — *hint:* what is the ball's velocity at that moment?) (Give your answer correct to 2 decimal places.)

16. A person standing 1.5 m above the ground throws a ball up in the air with an initial velocity of 9 m/sec. The position function is $x(t) = -4.9t^2 + v_0 t + x_0$, where v_0 is the initial velocity, and x_0 the initial position. What are the velocity and acceleration functions? At what time t does the ball hit the ground? At what time t does it "turn around"? (*I.e.* stop going up and start coming back down — *hint:* what is the ball's velocity at that moment?) (Give your answer correct to 2 decimal places.)