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NYA Cal I — Limits Workshop

1. Evaluate each of the following limits.

(a) $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{\sqrt{x}}\right)$ (b) $\lim_{x\to-\infty}\frac{\sqrt{x^2-3}}{4x+1}$ (c) $\lim_{x\to 0}\frac{\frac{1}{x+2}-\frac{2}{x+4}}{\sin(x)}$

(d) $\lim_{x\to 3}\frac{2-\sqrt{7-x}}{x^2-5x+6}$ (e) $\lim_{x\to 0}\frac{\ln(\mathrm{e}+h)-1}{h}$ (by interpreting it as a derivative)

(f) $\lim_{x\to 2}\frac{\frac{1}{x+4}-\frac{1}{3x}}{x-2}$ (g) $\lim_{x\to\frac{\pi}{3}^+}\frac{\sqrt{x}}{2\cos(x)-1}$ (h) $\lim_{x\to-\infty}\frac{9-3x}{2x-\sqrt{4x^2+3x-9}}$

(i) $\lim_{x\to\frac{1}{3}}\frac{9x^2-1}{8x-3|x^2-1|}$ (j) $\lim_{x\to 0^+}\mathrm{e}^{-5/x}\sin\left(\frac{5}{x}\right)$ (k) $\lim_{x\to-3}\frac{x^2+3x}{2x^2+5x-3}$

(l) $\lim_{x\to 2}\frac{\sqrt{2x+5}-3}{x^2-4}$ (m) $\lim_{x\to-\infty}\frac{5-8x^3}{(3x^2-2)(2x+3)}$ (n) $\lim_{x\to 0}\frac{\tan(3x)}{x}$

2. Find the horizontal and vertical asymptotes of $f(x)=\dfrac{3\,\mathrm{e}^x+1}{5\,\mathrm{e}^x-2}$.

3. Give the rule of a function of the form

$$f(x)=\frac{(Ax-B)(Cx-D)}{(Ex-F)(Gx-H)}$$

that has **all** of the following properties:

(a) $\lim_{x\to\infty} f(x)=1$. (b) $\lim_{x\to 3} f(x)$ exists, but $f(3)$ does not. (c) $\lim_{x\to 2^-} f(x)=\infty$.

4. Find the value of k that will make

$$f(x)=\begin{cases}\dfrac{4-x^2}{x^3+8} & \text{if } x<-2\\ x+k & \text{if } x\geq -2\end{cases}$$

continuous at $x=-2$.

5. Let

$$f(x)=\begin{cases} x^2+6x-18 & \text{if } x<b\\ a & \text{if } x=b\\ 5x-6 & \text{if } x>b.\end{cases}$$

Find all pairs of values a and b so that the function f is continuous everywhere.

Questions taken from exams of May2016, Dec2014, Dec2007. Answers may be found there!