Instructor: Dr. R.A.G. Seely — Test 3

(May 2019) — (Version C)

Cal I (S) (Maths 201–NYA)

(Marks)

Justify *all your answers—just having the correct answer is not sufficient.*
Pace yourself—a rough guide is to spend less than $2m$ minutes or so on a question worth m marks.

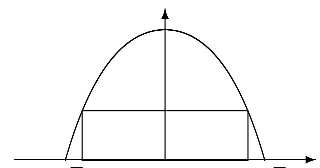
(6) 1. For a continuous function $y = f(x)$, suppose you know its derivative is postive for $-\infty < x < 1$ and for $3 < x < +\infty$, and that its derivative is negative for $1 < x < 3$. Furthermore suppose you know that its second derivative is positive for $-\infty < x < 0$ and $2 < x < 4$, and is negative for $0 < x < 2$ and $4 < x < +\infty$. Finally, suppose you know the x-axis is a horizontal asymptote and that $(2, 0)$ is the only x-intercept. Draw a rough sketch of the function showing all these properties.

(7) 2. For the following function

$$y = 2 + \frac{1}{x} - \frac{1}{x^2}$$

graph the function, identifying all intercepts, aymptotes, local extrema, and inflection points. Specify intervals where the graph is increasing, decreasing, concave up, and concave down. Show all your work.

(6) 3. A rectangle is to be inscribed inside the parabola $y = 16 - 2x^2$ so that its base sits on the x-axis, and its top vertices touch the parabola. Find the dimensions that give the rectangle with the largest possible area.



(3×5) 4. Evaluate the following:

(a) $\displaystyle\int_1^2 \left(x - \frac{1}{x}\right)^2 dx$

(b) $\displaystyle\int_0^{\frac{\pi}{4}} \tan^2\theta \; d\theta$

(c) $\displaystyle\int \left(\mathrm{e}^2 + \sqrt[5]{x^3} - \mathrm{e}^x\right) dx$

(5) 5. Given $f''(x) = 2 + \sin x$ satisfying $f'(\pi) = 2\pi,\ f(\pi) = 0$, find the function f.

(3) 6. For the function $f(x) = \displaystyle\int_0^{x^2} \frac{1}{1+t^2}\, dt$, what is the derivative $f'(x)$? What is the minimum value of $f(x)$?

(4) 7. The graph of $y = \frac{3}{4}x^4 - 6x^2 + 12$ has exactly two x-intercepts (at ± 2), and so there is a closed region between them, the curve, and the x-axis. Find the area of that region. *(You may—or may not!—find that sketching a very rough graph will help you solve this problem.)*

(4) 8. Express the integral $\displaystyle\int_0^2 x\, 2^{2x}\, dx$ as the limit of a Riemann sum. (You do not need to evaluate the limit!)

(Total: 50)