Instructor: Dr. R.A.G. Seely
(April 2019)

Test 2
(Version B)

Cal I (S) (Maths 201–NYA)

(Marks)

Justify *all your answers—just having the correct answer is not sufficient.*
Pace yourself—a rough guide is to spend less than $2m$ minutes or so on a question worth m marks.

(7×3) 1. Calculate the following limits (if they exist). If a limit does not exist, say so; if a limit is infinite, say so. If appropriate one-sided limits give more information, state that as well, and if they exist, give their values explicitly.

(a) $\lim\limits_{x\to 3} \dfrac{\sqrt{x+1}-2}{x-3}$ (b) $\lim\limits_{x\to+\infty} \left(\sqrt{4x^2-3x+5}-2x\right)$

(c) $\lim\limits_{x\to 4} \dfrac{3x-1}{x+4}$ (d) $\lim\limits_{x\to -4} \dfrac{3x-1}{x+4}$ (e) $\lim\limits_{x\to 4} \dfrac{\frac{3}{2x+1}-\frac{1}{3}}{x-4}$

(f) $\lim\limits_{x\to\infty} \dfrac{\sin 3x}{5x}$ (g) $\lim\limits_{x\to 0} \dfrac{\sin 3x}{5x}$

(3×3) 2. Find all horizontal and vertical asymptotes for the following functions. (Check both $x, y \to \pm\infty$.)

(a) $y = \dfrac{\sqrt{4x^2-9}}{2x-3}$ (b) $y = \dfrac{2x^2-5x+3}{3x^2-5x+2}$ (c) $y = \dfrac{7}{2+\mathrm{e}^x}$

(4) 3. For the function $f(x) = \begin{cases} a^3+a^2x+2 & \text{if } x<2 \\ a^2+ax+x & \text{if } x\geq 2 \end{cases}$
find all values of a that make $f(x))$ continuous at $x=2$.

(4) 4. For the function $f(x) = \begin{cases} \dfrac{2x^2-3x+1}{4x^2-1} & \text{if } x<\frac{1}{2} \\ \frac{1}{4} & \text{if } x\geq\frac{1}{2} \end{cases}$
find all the values of x for which the function is discontinuous. For each, specify if the discontinuity is removable or not. If it is removable, redefine the function at that point to remove the discontinuity.

(4) 5. A conical water tank is being drained at a constant rate. The tank is 15 m high and 6m in diameter (at its top). The water level is falling at a rate of 100 cm/min when the level is 5 m. Find the rate at which the tank is being emptied. (Hint: $V = \frac{1}{3}\pi r^2 h$)

(4) 6. A lighthouse is located on a small island 4 km away from the nearest point P on a straight shoreline; its light makes two revolutions per minute. How fast is the beam of light moving along the shoreline when it is 2 km from P?

(4) 7. Find the values of x where the absolute (or global) minimum and maximum values of the function $f(x) = \dfrac{x}{2+x^2}$ occur on the interval $[-3,3]$ (*i.e.* $-3 \leq x \leq 3$).

(Total: 50)