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Test 2
(Version B)

Cal I (S) (Maths 201–NYA)

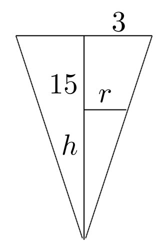
Answers

1. (a) $1/4$ (b) $-3/4$ (c) $11/8$
   (d) DNE: Left $\lim = +\infty$, Right $\lim = -\infty$
   (e) $-2/27$ (f) 0 (g) $3/5$

2. (a) HAs: $y = \pm 1$; VA: $x = \frac{3}{2}$. (b) HA: $y = \frac{2}{3}$; VA: $x = \frac{2}{3}$. (c) HAs: $y = 0, \frac{7}{2}$; no VA.

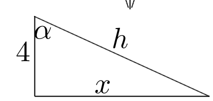
3. $a = 0, 1, -2$

4. Discontinuous at $x = -1/2$ (not removable: $\lim = \pm\infty$), and at $x = 1/2$ (a "gap discontinuity"; note that the limit $\lim\limits_{x \to 1/2} f(x)$ does not exist, since the left and right limits are not equal, so this is not a removable discontinuity).



5. $\frac{r}{h} = \frac{3}{15}$, $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \frac{1}{25} h^3$, so $\frac{dV}{dt} = \frac{\pi}{25} h^2 \frac{dh}{dt}$.
   When $h = 5$m, $\frac{dh}{dt} = -1$ m/min, so $\frac{dV}{dt} = \frac{\pi}{25} \cdot 5^2 \cdot (-1) = -\pi$.
   So the tank empties at π m^3/min.

6. Let α be the angle as shown:
   $\frac{d\alpha}{dt} = 4\pi$; $\frac{x}{4} = \tan\alpha$, so $\frac{1}{4}\frac{dx}{dt} = \sec^2\alpha \, \frac{d\alpha}{dt}$, so
   $\frac{dx}{dt} = 4(\frac{\sqrt{20}}{4})^2 \cdot 4\pi = 20\pi$ km/min if $x = 2$.



7. $f'(x) = \dfrac{2 - x^2}{(2 + x^2)^2}$: Max at $x = \sqrt{2}$; min at $x = -\sqrt{2}$.
   Note: The critical points are $\pm\sqrt{2}$, so the max and min are at some of $x = \pm 3, \pm\sqrt{2}$; the trick is to decide which is which, so we compare the values at those four points: we get $\pm\frac{3}{11}$ and $\pm\frac{\sqrt{2}}{4}$. But we can estimate these values: $\frac{3}{11} < \frac{3}{10} = .3$, but $\frac{\sqrt{2}}{4} \simeq \frac{1.4}{4} = .35$, so $\frac{\sqrt{2}}{4} > \frac{3}{11}$.
   Alternatively, we can use the shape of the graph, using the values of the derivative: $f'(x) = \frac{2-x^2}{(2+x^2)^2} < 0$ on $[-3, -\sqrt{2}]$, so f gets smaller on that domain (so the min is at $-\sqrt{2}$), $f(x) > 0$ on $[-\sqrt{2}, \sqrt{2}]$, so increases (to the max at $\sqrt{2}$), and then $f(x) < 0$ on $[\sqrt{2}, 3]$, so f gets smaller there: the max is at $\sqrt{2}$.