



Cal I (S) (Maths 201–NYA)

(Marks)

Justify all your answers—just having the correct answer is not sufficient.

Pace yourself—a rough guide is to spend less than 2m minutes or so on a question worth m marks.

- (7 × 3) 1. Calculate the following limits (if they exist). If a limit does not exist, say so; if a limit is infinite, say so. If appropriate one-sided limits give more information, state that as well, and if they exist, give their values explicitly.

(a) $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3}$

(b) $\lim_{x \rightarrow +\infty} (\sqrt{4x^2 - 3x + 5} - 2x)$

(c) $\lim_{x \rightarrow 4} \frac{3x - 1}{x + 4}$

(d) $\lim_{x \rightarrow -4} \frac{3x - 1}{x + 4}$

(e) $\lim_{x \rightarrow 4} \frac{\frac{3}{2x+1} - \frac{1}{3}}{x - 4}$

(f) $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$

(g) $\lim_{x \rightarrow \infty} \frac{\sin 3x}{5x}$

- (3 × 3) 2. Find all horizontal and vertical asymptotes for the following functions. (Check both $x, y \rightarrow \pm\infty$.)

(a) $y = \frac{2x^2 - 5x + 3}{3x^2 - 5x + 2}$

(b) $y = \frac{\sqrt{4x^2 - 9}}{2x - 3}$

(c) $y = \frac{7}{2 + e^x}$

- (4) 3. For the function $f(x) = \begin{cases} \frac{2x^2 - 3x + 1}{4x^2 - 1} & \text{if } x < \frac{1}{2} \\ \frac{1}{4} & \text{if } x \geq \frac{1}{2} \end{cases}$

find all the values of x for which the function is discontinuous. For each, specify if the discontinuity is removable or not. If it is removable, redefine the function at that point to remove the discontinuity.

- (4) 4. For the function $f(x) = \begin{cases} a^3 + a^2x + 2 & \text{if } x < 2 \\ a^2 + ax + x & \text{if } x \geq 2 \end{cases}$

find all values of a that make $f(x)$ continuous at $x = 2$.

- (4) 5. A lighthouse is located on a small island 2 km away from the nearest point P on a straight shoreline; its light makes two revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P ?

- (4) 6. A conical water tank is being drained at a constant rate. The tank is 15 m high and 6m in diameter (at its top). The water level is falling at a rate of 100 cm/min when the level is 5 m. Find the rate at which the tank is being emptied. (Hint: $V = \frac{1}{3}\pi r^2 h$)

- (4) 7. Find the values of x where the absolute (or global) minimum and maximum values of the function $f(x) = \frac{x}{2 + x^2}$ occur on the interval $[-3, 3]$ (i.e. $-3 \leq x \leq 3$).

(Total: 50)